Math 185 – Quiz #3

November 21, 2007 **Name:** Solution Set
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Total: 40 points

1. The sparks from an emery wheel used to sharpen blades fly off tangent to the wheel. Find the equation along which sparks fly from a wheel described by $x^2 + y^2 = 25$ at the point (3, 4). (6 points)

implicit diff:

$\frac{d}{dx}(x^2+y^2) = \frac{d}{dx}(25)$

$2x + 2y\frac{dy}{dx} = 0$ (1)

$2y\frac{dy}{dx} = -2x$

$\frac{dy}{dx} = \frac{-x}{y}$ (1)

$\left.\frac{dy}{dx}\right|_{(3,4)} = -\frac{3}{4}$ (1)

$m = -\frac{3}{4}$, $(3,4)$ (1)

$y - y_1 = m(x - x_1)$

$y - 4 = -\frac{3}{4}(x-3)$ (1)

$y = -\frac{3}{4}x + \frac{9}{4} + 4$

$\boxed{y = -\frac{3}{4}x + \frac{25}{4}}$ slope-intercept

or (1)

$4y = -3x + 25$

$\boxed{3x + 4y = 25}$ standard form

2. Calculate the differential dy for the following functions. (6 points)

a) $y = \sqrt{x^2-1} = (x^2-1)^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}(x^2-1)^{-1/2}\, 2x = \frac{x}{\sqrt{x^2-1}}$ (2)

$dy = \frac{dy}{dx}dx = \frac{x}{\sqrt{x^2-1}}dx$ (1)

$dy = \frac{x}{\sqrt{x^2-1}}dx$ (3)

b) $y = \frac{x}{x+1}$

$dy = \frac{1}{(x+1)^2}dx$ (3)

$\frac{dy}{dx} = \frac{(x+1)\cdot 1 - x\cdot 1}{(x+1)^2}$

$= \frac{x+1-x}{(x+1)^2}$

$= \frac{1}{(x+1)^2}$

$dy = \frac{dy}{dx}dx$

$= \frac{1}{(x+1)^2}dx$

3. A radio-controlled model car is operated in a parking lot. The coordinates of the car (coordinates in metres) are given by $x = 4 - 3t^2$ and $y = \frac{16}{t}$, where t is the time in seconds. Find the **magnitude** of the **acceleration** of the car after 2 seconds. You may leave your answer in radical form. (6 points)

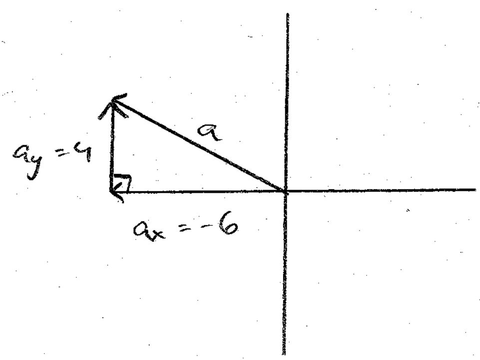
$x = 4 - 3t^2$ $\quad\quad$ $y = 16t^{-1}$

$v_x = \frac{dx}{dt} = -6t$ 1 $\quad\quad$ $v_y = \frac{dy}{dt} = -16t^{-2}$ 1

$a_x = \frac{d^2x}{dt^2} = -6$ 1 $\quad\quad$ $a_y = \frac{d^2y}{dt^2} = 32t^{-3} = \frac{32}{t^3}$ 1

at $t = 2$ seconds

$a_x = -6$ $\quad\quad$ $a_y = \frac{32}{2^3} = \frac{32}{8} = 4$ 1



$$a = \sqrt{a_x^2 + a_y^2}$$
$$= \sqrt{(-6)^2 + (4)^2}$$
$$= \sqrt{36 + 16}$$
$$= \sqrt{52}$$
$$= 2\sqrt{13}$$ 1

$$\boxed{a = 2\sqrt{13}\ m/s^2}$$

The magnitude of the acceleration is $2\sqrt{13}$ m/s^2.

4. Consider the function $y = x^3 - 6x^2$.

a) Find the x- and y- intercepts, if any. (3 points)

y-int : ~~set~~ $x = 0$ $y = 0^3 - 6 \cdot 0^2 = 0$

y-int : $(0,0)$ 1

x-int : set $y = 0$

$0 = x^3 - 6x^2$

$0 = x^2(x-6)$

$x = 0, 6$

x-int : $(0,0)$ and $(6,0)$ 2

b) Find any critical values, if any. If there are critical points, use one of the derivative tests to verify whether these points are relative maxima or minima and show your work. (6 points)

$0 = \frac{dy}{dx} = 3x^2 - 12x$ 1

$0 = 3x(x-4)$

$x = 0, 4$ 1

$y = x^3 - 6x^2$

$\frac{dy}{dx} = 3x^2 - 12x$

$\frac{d^2y}{dx^2} = 6x - 12$ 1

When $x = 0$, $y = 0$

at $(0,0)$

$\frac{d^2y}{dx^2} = -12$ so negative

$\therefore$ concave down

$\rightarrow$ rel. maximum at $(0,0)$

1

when $x = 4$, $y = x^2(x-6)$

$= 16(-2)$

$= -32$

at $(4,-32)$ 1

$\frac{d^2y}{dx^2} = 6x - 12$

$= 24 - 12$

$= +$

$\therefore$ concave up

$\rightarrow$ rel. minimum at $(4,-32)$

1

c) Find the inflection points of this function, if any. (3 points)

$$\frac{d^2y}{dx^2} = 6x - 12 = 0 \quad 1$$

$$6x = 12 \quad 1$$

$$x = 2$$

when $x = 2$, $y = x^2(x-6)$

$$= 4(-4) \quad 1$$

$$= -16$$

$(2, -16)$ is an inflection point

d) Sketch the resulting graph. (4 points)

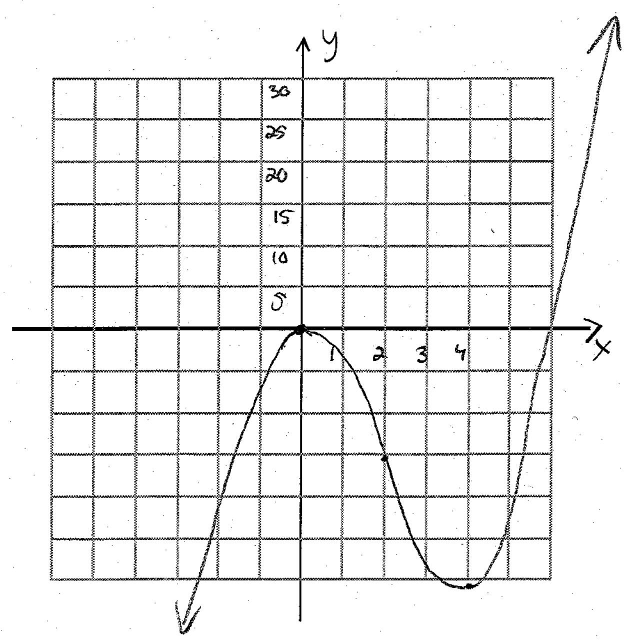
end behaviour:

leading term is x^3,

so graph will end up

as ↗

↙



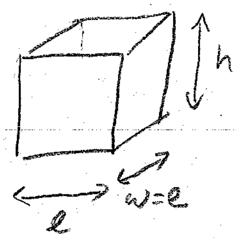
graph: correct end behaviour 1

correct intercepts 1

max/min behaviour 1

inflection 1

5. A rectangular box is open at the top and has a square base. The box is designed to have a volume of 4 m³. What are the dimensions required if you want to minimize the amount of cardboard needed to build this box? (6 points)



$V = lwh$

$4 = l^2 h$

$h = \frac{4}{l^2}$ 1

missing top

↓

$A = 2lh + 2wh + lw$ 1

$A = 2lh + 2lh + l^2$ since $l = w$

$A = 4lh + l^2$

$= 4l\left(\frac{4}{l^2}\right) + l^2$

$A = \frac{16}{l} + l^2$ 1

$\frac{dA}{dl} = -\frac{16}{l^2} + 2l = 0$

$2l = \frac{16}{l^2}$

$l^3 = 8$

$l = 2$ 1

truly a min?

$\frac{d^2A}{dl^2} = \frac{32}{l^3} + 2$ 1

$= +$ for $l = 2$

concave up

$\therefore$ min ✓

$h = \frac{4}{l^2} = 1$

The dimensions are 1

2m × 2m for the base

and 1m for the height