Section V.1: Vectors and Scalars

Introduction

Suppose you go for a walk downtown, walking a total distance of 2 blocks. How far are you from your original location? It will depend on whether you walked the 2 blocks in a straight line or turned left or right after the first block, so it is important to know not only how far you walked, but also in which direction(s). Also, if someone asked you where you were with respect to your original location, you'd probably give both a distance and a direction in your answer. In math (and physics and engineering), we make a distinction between quantities that have a size associated with them and quantities that have both a size and a direction. The technical term for the size of these quantities is the **magnitude**.

Definitions

scalar – a quantity that has magnitude only

vector – a quantity that has both magnitude and direction

In both cases, the magnitude (size) can be expressed as a single real number, which may include units. For example, "5 years", "8.3 km", and "0.2 kg" are all scalars. However, the quantity "8.3 km north" is a vector which has both a magnitude (8.3 km) and a direction (north).

There are many quantities in physics that are vectors. For example, velocity, displacement, acceleration, force, and magnetic field strength are all vectors, to name just a few. Vectors may also be used in other fields. The geometric modeling used in computer animations uses a vast number of vectors. Vectors may also be used in the study of economics (to track quantities and prices) and biology (to track aspects of populations).

Example

Which of the following are vectors, scalars, or neither?

a) 5.2 m/s left
b) downwards
c) 0.52 s
d) 15º south of east

Answer:

a) vector, because has both magnitude and direction
b) neither a vector nor a scalar, because has no magnitude, only direction
c) scalar, because has magnitude only

d) neither, because it is a direction only (Careful! You might think that the 15º is a magnitude, but it is only specifying the direction more exactly, in the same way that "north" specifies an exact direction.)

Notation

There are two ways to distinguish vectors from scalars when writing them in variable form. If the vector is handwritten, the variable symbol is written with a right-pointing arrow over top: $\vec{F}$. Even if the vector itself is pointing left or down or northeast, the arrow on top of the symbol still points right. Also, the bottom half of the arrow's tip is usually omitted, as shown above. If, however, the vector appears in a textbook or other typeset document, the variable is usually bolded: **F**.

Example

An object is northeast of its initial position and a distance of 5.3 m away. Using vector notation, the object's **displacement** is given by

handwritten: $\vec{d}$ = 5.3 m northeast

typeset: **d** = 5.3 m northeast

Suppose we wished to talk about the magnitude of the vector given in the previous example, 5.3 m. There are two acceptable ways to symbolize this. We can write the vector symbol with absolute value bars around it, like so: $\left|\vec{d}\right|$ or |**d**|, depending on whether the vector is handwritten or typeset. We also could just use the symbol "d", without the arrow/bolding.

Example

What symbol could you use for the magnitude of the force $\mathbf{F_A}$?

Answer:

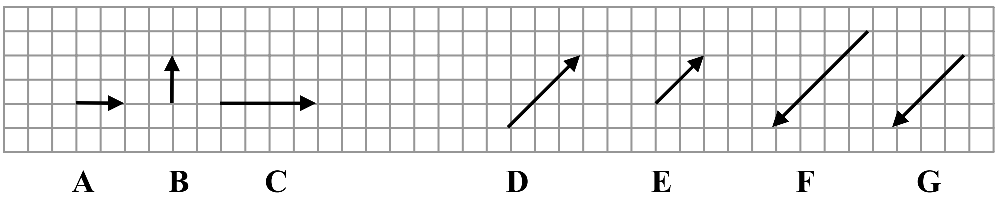
Either $|F_A|$ or F_A could be used.

Note: writing the magnitude with single absolute value bars, $\left|\vec{d}\right|$, is the notation used in science and engineering. Should you encounter vectors in advanced math courses, the magnitude is usually symbolized with double bars, $\left\|\vec{d}\right\|$. To be consistent with your other courses in the technology program, we will stick with the science notation for the purposes of this course.

Geometric representation

To represent a vector in a diagram, we draw an arrow. The length of the arrow corresponds to the magnitude of the vector, and the direction of the arrow should be the same as the direction specified for the vector.

Consider the vectors in the diagram below.



Vector **B** has the same magnitude as **A**, since it has the same length of 2 units on the grid. Vector **C** has the same direction as **A** but a different magnitude, since it is 3 units long.

If we wished to write the above information as equations, we could write that

$$|\mathbf{A}| = |\mathbf{B}|$$

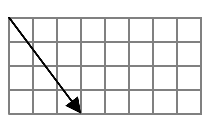
and

$$|\mathbf{A}| \neq |\mathbf{C}|.$$

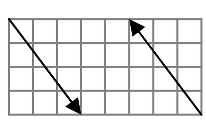
Similarly, **E** has the same direction as **D** but a smaller magnitude, while **F** and **G** are in the opposite direction to **D**. In fact, **G** is said to be the opposite or negative of **D**, because it has the same magnitude, but the opposite direction.

Example

Consider the vector **V** on the grid below. Beside it, sketch the vector that is opposite to **V**.



Answer:



The Zero Vector

The zero vector has a magnitude of zero. Because it has no length, this is the only vector that doesn't have a direction associated with it.