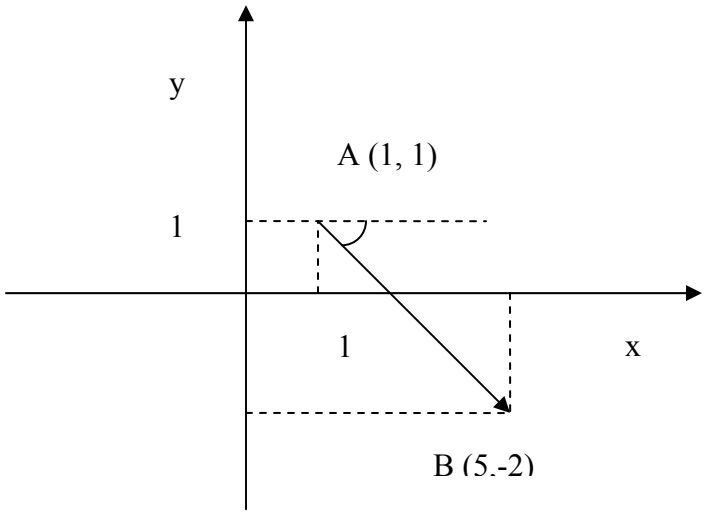
Section V.2: Magnitudes, Directions, and Components of a Vector

Exercise Solutions

Describe the direction of the following vectors, using angles in standard position. Round any angles to one decimal place.

1. The vector that runs from the point (1,1) to (5,–2).



$\vec{AB} = (4,-3) \Rightarrow \text{AB} = \sqrt{(5-1)^2 + (-2-1)^2} = 5 \quad \tan^{-1}(-\frac{3}{4}) = -36.9^\circ$

$360^\circ - 36.9 = 323.1^\circ$

2. The vector that runs from the point (5,-2) to (1,1).

$\vec{BA} = (-4,3) \Rightarrow \text{AB} = \sqrt{(5-1)^2 + (-2-1)^2} = 5 \qquad \alpha = \tan^{-1}(-\frac{3}{4}) = -36.9^\circ$

$\theta = 180^\circ - 36.9 = 143.1^\circ$

3. Compare your answers for #1 and #2. Why are they the same/different?

The 2 vectors are opposite

State the magnitude of the following vectors. Give exact answers.

4. The vector that runs from the point (1, 1) to (5,–2).

$\vec{AB} = (5-1,-2-1) = (4,-3) \Rightarrow$ AB =5

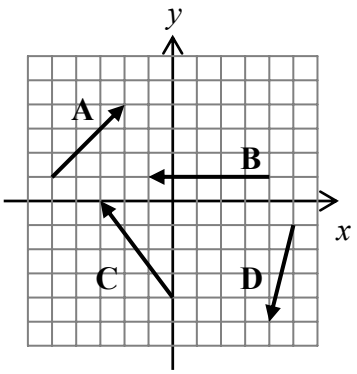
5. The vector that runs from the point (5,–2) to (1, 1).

V$= (1-5,1+2) = (-4,3)$ $\quad |\boldsymbol{V}| = 5$

6. Compare your answers for # 4 and # 5. Why are they the same/different?

The 2 vectors are opposite: same magnitude, opposite directions

Consider the four vectors in the diagram below.



7. Give the magnitude and direction (in standard position) for these vectors. Give exact answers for the magnitudes and round any angles to the nearest degree.

|**A**|$= \sqrt{3^2+3^2} = \sqrt{3^2+3^2} = 3\sqrt{2}$ $\quad \theta = \tan^{-1} 1 = 45^\circ$

| **B**| $= 5$ $\quad \theta = 180^\circ$

|**C**|$= 5$ $\quad \theta = 180^\circ + \tan^{-1}(-\frac{4}{3}) = 180 - 53.1^\circ = 126.9^\circ$

|**D**|$= \sqrt{17}$ $\quad \theta = 180 + \tan^{-1} 4 = 256^\circ$

8. Write the component form of each vector in the above diagram.

A=3**i**+3**j**; **B= -5j;C=-3i+4j; D= -i-4j**

State the magnitude and direction of the following vectors. Give exact answers.

9. $B_x = 35, B_y = 0$ B=35

10. $F_x = -16, F_y = -16$ $F=16\sqrt{2}$

11. $\mathbf{A} = 3\mathbf{i} - 3\sqrt{3}\,\mathbf{j}$ A = 6

12. $\mathbf{A} = -7\mathbf{j}$ A=7

State the magnitude and direction of the following vectors. Round your answers to the nearest decimal place.

13. $A_x = 2.3$ m, $A_y = 5.4$ m $A=\sqrt{2.3^2 + 5.4^2} = 5.9$ $\theta = \tan^{-1}\frac{A_y}{A_x} = 66.9^\circ$

14. $E_x = -16, E_y = 17$ $E=\sqrt{16^2 + 17^2} = 23.3$ $\theta = 180 + \tan^{-1}\frac{17}{-16} = 133.3^\circ$

15. $\mathbf{A} = 1.1\mathbf{i} - 2.7\mathbf{j}$ $\mathbf{A}=\sqrt{1.1^2 + 2.7^2} = 2.9$ $\theta = 360^\circ + \tan^{-1}(\frac{-2.7}{1.1}) = 292.2^\circ$

16. $\mathbf{A} = -6.5\mathbf{i}$ A= 6.5 $\theta = 180^\circ$

Give the component form of the following vectors. Give exact answers in **ijk** notation.

17. A = 5, θ = 45º $\mathbf{A}=5\frac{\sqrt{2}}{2}\mathbf{i} + 5\frac{\sqrt{2}}{2}\mathbf{j}$

18. A = 3, θ = 90º **A= 0i + 3j**

19. A = 2, θ = 120º **A=(**2 cos120º)i+(2sin120º)**j= - i +** $\sqrt{3}$ **j**

20. A = 7, θ = 0 A=7**i** + 0**j**

Give the component form of the following vectors in the form A_x and A_y. Round answers to one decimal place.

21. A = 5, θ = 22º **A** = (5cos22º) **i** + (5sin22º)**j** =4.6 **i** +1.9**j**

22. A = 3, θ = –100º **A** = (3cos–100º) **i** + (3sin–100º)**j= -**0.5**i –** 3.0**j** (Pat notes that this looks wrong, because the **j**-component has been rounded from 2.95 up to 3.0, so it looks like the component is as big as the hypotenuse. What it means is that the *y*-component is almost as big as the hypotenuse, and when you round it, the values become the same within sigfigs. Sorry!)

Find a unit vector **u** in the same direction as the given vector. Give exact answers.

23. $\mathbf{A} = 4\mathbf{i} - 3\mathbf{j}$ $= \vec{u} = \frac{\vec{A}}{A} = \frac{4}{5}\vec{i} - \frac{3}{5}\vec{j}$

24. $\mathbf{A} = 7\mathbf{i}$ $\vec{u} = \vec{i}$

25. $\mathbf{A} = -2\mathbf{i} + 2\mathbf{j}$ $\vec{u} = \frac{\vec{A}}{A} = -\frac{\sqrt{2}}{2}\vec{i} + \frac{\sqrt{2}}{2}\vec{j}$

26. $\mathbf{A} = -7\mathbf{i} - 24\mathbf{j}$ $\vec{u} = \frac{\vec{A}}{A} = -\frac{7}{25}\vec{i} - \frac{24}{25}\vec{j}$

27. $\mathbf{A} = \mathbf{i} - \sqrt{3}\,\mathbf{j}$ $\vec{u} = \frac{\vec{A}}{A} = \frac{1}{2}\vec{i} - \frac{\sqrt{3}}{2}\vec{j}$

28. A = 5, θ = 22º (hint: don't think too hard on this one)

$$\vec{u} = \frac{\vec{A}}{A} = (5\cos 22^\circ)\,\vec{i} - (5\sin 22^\circ)\,\vec{j} = 4.6\,\vec{i} + 1.9\,\vec{j}$$

Given that $\mathbf{A} = 3\mathbf{i} - 4\mathbf{j}$ and **B** is a vector with magnitude of 3 and direction 135º in standard position, calculate the following. Leave your answers in whatever form you prefer (it's easiest to leave them in the form of the original vector).

29. $5\mathbf{A} = 5(3\mathbf{i} - 4\mathbf{j}) = 15\mathbf{i} - 20\mathbf{j}$

30. $-\mathbf{A} = (-1)(3\mathbf{i} - 4\mathbf{j}) = -3\mathbf{i} + 4\mathbf{j}$

31. $-\mathbf{B} = (-1)\left(-\frac{3\sqrt{2}}{2}\mathbf{i} + \frac{3\sqrt{2}}{2}\mathbf{j}\right) = \frac{3\sqrt{2}}{2}\mathbf{i} - -\frac{3\sqrt{2}}{2}\mathbf{j}$

32. $3\mathbf{B} = -\frac{9\sqrt{2}}{2}\mathbf{i} + \frac{9\sqrt{2}}{2}\mathbf{j}$

33. What's the terminal point of the vector $\mathbf{B} = -2\,\mathbf{i} - 3\,\mathbf{j}$ which starts at the point (2,4)?

Answer: (0, 1)

34. What's the initial point of the vector $\mathbf{B} = 5\,\mathbf{i} - 8\,\mathbf{j}$ which ends at the point (3,–1)?

Answer: (-2, 7)

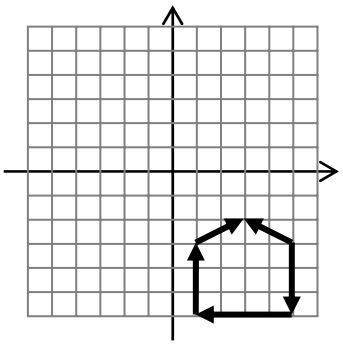
35. Write in **ijk** notation the vector that runs from the point (1,1) to (5,–2).

$(x_{final} - x_{initial})$ i+ $(y_{final} - y_{initial})$ j + $(z_{final} - z_{initial})$ **k**= (5-1) **i** + (-2-1) **j**+ 0 **k** = 4 **i**– 3 **j**

Sketch the following vectors on the accompanying grid.

A = 3**j**, starting at the point (1,–6) End point at (1,-3)

36.



37. **B** = –2**i** + **j**, starting at the point (5,–3) End point at (3,-2)

38. **C** = –3**j**, starting at the point (5,–3) End point at (5,-6)

39. **D** = –4**i**, starting at the point (5,–6) End point at (1,-6)

40. **E** = 2**i** + **j**, starting at the point (1,–3) End point at (3,-2)