Section V.3: Dot Product

Exercise Solutions

Find the dot product **A·B** of the following vectors.

1. **A** has components $A_x = -3$, $A_y = 6$; **B** has components $B_x = -5$, $B_y = -6$.

A·B=(-3)(-5) + 6(-6)=15-36 = -21

2. **A** has components $A_x = 17$, $A_y = 34$; **B** has components $B_x = 16$, $B_y = -8$.

A·B=17 · 16 + 34 · (-8) = 0

3. **A** = 3**i**, **B** = **j** **A·B** = 0

4. **A** = **i**, **B** = 2**i** **A·B** = 2

5. **A** = 3**i** + **j**, **B** = 7**i** – 2**j** **A·B** = 19

6. **A** = 4**i** – 3**j**, **B** = **i** **A·B** = 4

7. **A** = –3**i** + 2**j**, **B** = –8**j** **A·B**= -16

8. **A** = **i** + **j**, **B** = 2**i** – 2**j** **A·B** = 0

9. **A** = 3**i** + **j** –2**k**, **B** = 2**i** + 3**k** **A·B** = 0

10. **A** = 12**i** – 9**j** – 10**k**, **B** = 3**i** + **j** – 4**k** **A·B** = 67

11. **A** = 3 units at 45º, **B** = 4 units at 210º **A·B** = 12 cos (210 º -45 º) = -11.6

12. **A** = 4.5 units at –15º, **B** = 10 units at 345º **A·B** = 45

13. **A** = 2 units at –60º, **B** = –3**i** – 3**j** $\mathbf{A} = \mathbf{i} - \sqrt{3}\,\mathbf{j}$; $\mathbf{A\cdot B} = -3 + 3\sqrt{3}$

14. **A** = 7**i**, **B** = 4 units at 150 $\mathbf{B} = -2\sqrt{3}\,\mathbf{i} + 2\mathbf{j}$ $\mathbf{A\cdot B} = -14\sqrt{3}$

or **A·B**=28cos 150 º $= -14\sqrt{3}$

Calculate the magnitude of the following vectors using the dot product.

15. **A** has components $A_x = -3$, $A_y = 6$ $A = 3\sqrt{5}$

16. **B** has components $B_x = 16$, $B_y = -8$. $B = 8\sqrt{5}$

17. **A** = 7**i** – 24**j** A= 25

18. **D** = 5**i** + 8**j** D = $\sqrt{89}$

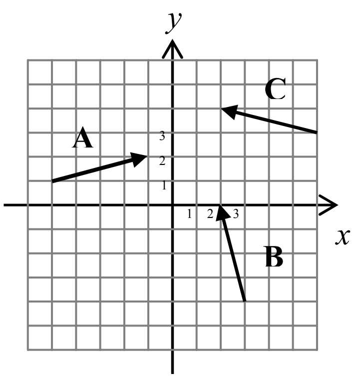
19. **F** = –8**i** – 12**j** F = $4\sqrt{13}$

20. **W** = 15**i** – 8**j** W = 17

21. **N** = 3**i** + **j** –2**k** N = $\sqrt{14}$

22. **A** = 12**i** – 9**j** – 10**k** A = $5\sqrt{13}$

23. Using the vectors in the diagram below, calculate **A·B, A·C,** and **B·C**.



A= 4**i**+**j** **B= -i**+4**j** **C= -** 4**i**+**j**

A·B = 0 **A·C**= -15 **B·C**= 8

Are the following pairs of vectors perpendicular? Use the dot product to determine your answer.

24. **A** has components A_x = 4, A_y = 7; **B** has components B_x = –7, B_y = –4. **No**

25. **A** = 3**i** + **j**, **B** = 7**i** – 2**j** **No**

26. **A** = 5**i** + 3**j**, **B** = 5**i** – 3**j** **No**

27. **A** = 5**i** + 3**j**, **B** = 3**i** – 5**j** **Yes**

28. **A** = 3**i** + **j** – 2**k**, **B** = 7**i** – 2**j** + **k** **No**

29. **A** = 5**i** – 3**j** + 4**k**, **B** = –2**i** – 2**j** + **k** **Yes**

30. Using your answer for # 23, are any of these pairs of vectors perpendicular?
A and **B**

Find the angle between each pair of vectors.

31. **A** = 3**i** + **j**, **B** = **i** – 2**j** **AB =**AB cos θ ∴ $\cos\theta = \frac{A_x B_x + A_y B_y}{AB}$

$\cos\theta = \frac{1}{5\sqrt{2}}$, $\theta = \cos^{-1}(\frac{1}{5\sqrt{2}}) = 81.9^\circ$

32. **A** = 3**i**, **B** = 7**i** – 6**j** $\theta = \cos^{-1}(\frac{7}{\sqrt{85}}) = 40.6^\circ$

33. **A** = **i** + **j + k**, **B** = 2**i** – **j** – 3**k** $\theta = \cos^{-1}\frac{-2}{\sqrt{42}} = 108^\circ$

34. **A** = **i** + **k**, **B** = **j** – **k** $\theta = \cos^{-1}(\frac{-1}{2}) = 120^\circ$

35. **A** = 2**i** + **j** – 3**k**, **B** = –6**i** – 3**j** + 9**k** $\theta = \cos^{-1}\frac{-42}{42} = 180^\circ$