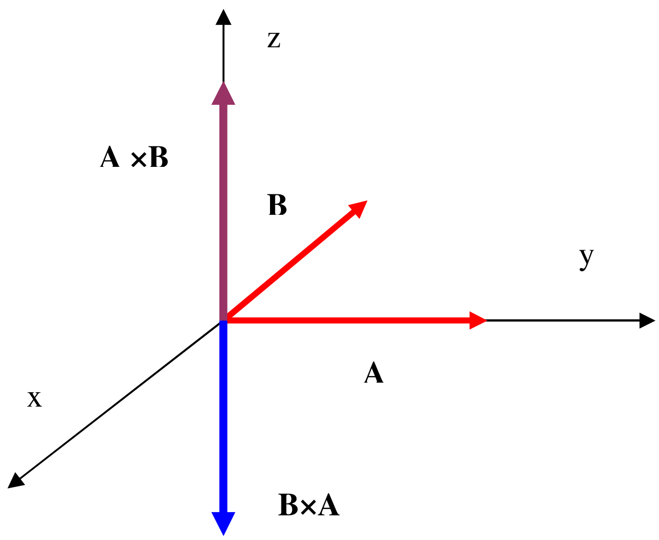
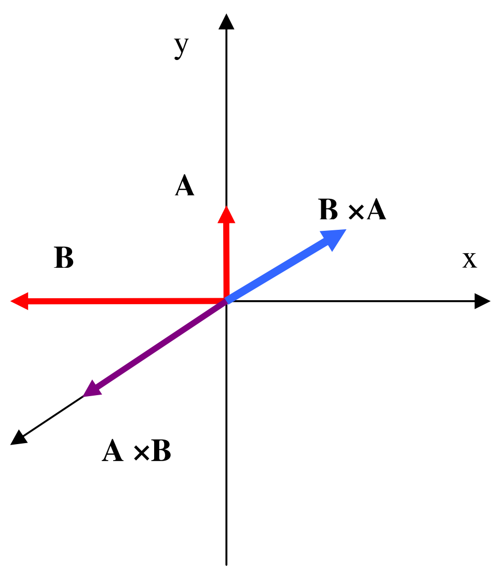
Section V.4: Cross Product

Solutions

1. Vector **A** is in the *y*-direction, while vector **B** is in the negative *x*-direction. What is the direction of **A×B**? **B×A?**





2. Vector **A** is in the *z*-direction, while vector **B** is in the *y*-direction. What is the direction of **A×B**? **B×A?**

A×B is in negative x- direction; **B×A** in (positive) x- direction.

Calculate the cross product **A×B** for the following vectors.

3. **A** = **i** , **B** = **j** **i×j = k**

4. **A** = **j** , **B** = **i** **j×i = - k**

5. **A** = **i** , **B** = **k** **i×k = -j**

6. **A** = **k**, **B** = **i** **k×i =j**

7. **A** = **k**, **B** = **j** **k×j = - i**

8. **A** = **j**, **B** = **j** **j×j= 0**

9. **A** = 2**i** – 9**j** – **k**, **B** = 3**i** + **j** – 4**k**

$$\mathbf{A\times B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -9 & -1 \\ 3 & 1 & -4 \end{vmatrix}$$

$$\mathbf{A\times B} = \vec{i}\begin{vmatrix} -9 & -1 \\ 1 & -4 \end{vmatrix} - \vec{j}\begin{vmatrix} -9 & -1 \\ 1 & -4 \end{vmatrix} + \vec{k}\begin{vmatrix} 2 & 9 \\ 3 & 1 \end{vmatrix} = \vec{i}(36+1) - \vec{j}(-8+3) + \vec{k}(2+27)$$

$$\mathbf{A\times B} = 37\,\vec{i} + 5\,\vec{j} + 29\,\vec{k}$$

10. **A** = 12**i** – 5**k**, **B** = 3**i** + **j** – 4**k** **A×B=**5**i**+33**j**+12**k**

11. **A** = **k**, **B** = 3**i** + 2**j** – 7**k** **A×B= -**2**i+**3**j**

12. **A** = 2**i** – **j**, **B** = 2**i** – **k** **A×B = i+**2**j+**2**k**

13. **A** = 5**i** +6**j** – 7**k**, **B** = **i** + 12**j** – 2**k** **A×B =** 72 **i+** 3**j+** 54 **k**

14. | **A×B** | = $\sqrt{72^2+3^2+54^2} = 90.05$

15. a) θ = 141.6º

 b) **A** = – 9**j** – 4**k**, **B** = 3**i** + 5**j**

 A×B= 20**i** -12**j** +27**k**

$|\mathbf{A}\times\mathbf{B}| = \sqrt{1273}$ A= $\sqrt{97}$ B=5 $\sqrt{34}$

And here's where you have to be careful! It's true that

$\sin\theta = \frac{|\mathrm{A}\times\mathrm{B}|}{AB}$ but that doesn't mean that $\theta = \sin^{-1}\frac{|\mathrm{A}\times\mathrm{B}|}{AB} = 38.4^{\circ}$ (?!) Why not? Recall that the $\sin^{-1}$ function has a range of [-90°, 90°], so by using the second expression you are restricting what possible answers you can get out of it. Recall also that sin θ = sin(90° – θ), so your answer should really not only include the Quadrant I answer but also the Quadrant II answer. So θ should really equal either 38.4° or the supplementary angle, 141.6°. Annoying, eh? Particularly when the cross product gives you no indication of which one is correct. That's why the dot product is the better method: taking the arccosine will give you answers in QI or QII, so you can just read the number off your calculator and not worry about having to make a correction.