Formula Sheet for the Final Exam

$$\int \tan x\,dx = \ln|\sec x| + C \qquad \int \cot x\,dx = -\ln|\csc x| + C$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C \qquad \int \csc x\,dx = -\ln|\csc x + \cot x| + C$$

$$\int \sec^2 x\,dx = \tan x + C \qquad \int \csc^2 x\,dx = -\cot x + C$$

$$\int \sec x\,\tan x\,dx = \sec x + C \qquad \int \csc x\,\cot x\,dx = -\csc x + C$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C \qquad \int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int u\,dv = uv - \int v\,du$$

Trapezoidal Rule: $\int_a^b f(x)\,dx \approx \frac{\Delta x}{2}\left(y_0 + 2y_1 + 2y_2 + 2y_3 + \cdots + 2y_{n-1} + y_n\right)$

Simpson's Rule: $\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \cdots + 4y_{n-1} + y_n\right)$

$$\sqrt{a^2 - x^2} \longrightarrow x = a\sin\theta, \qquad \sqrt{a^2 + x^2} \longrightarrow x = a\tan\theta, \qquad \sqrt{x^2 - a^2} \longrightarrow x = a\sec\theta$$

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

$$\sec\theta = \frac{1}{\cos\theta}, \qquad \csc\theta = \frac{1}{\sin\theta}, \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\cos^2\theta = \frac{1 + \cos 2\theta}{2} \qquad \sin^2\theta = \frac{1 - \cos 2\theta}{2} \qquad \sin 2\theta = 2\sin\theta\cos\theta$$

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots, \quad -1 < x \le 1$$

$$(1+x)^k = 1 + kx + \frac{k(k-1)x^2}{2!} + \frac{k(k-1)(k-2)x^3}{3!} + \cdots, \quad -1 < x < 1$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots, \quad \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots, \quad e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

Volume

disk: $dV = \pi r^2 dt$

shell: $dV = 2\pi r\, h\, dt$

Centroid/Centre of Mass

thin, uniform plate: $\bar{x} = \frac{1}{A}\int_A x_e\, dA$ $\bar{y} = \frac{1}{A}\int_A y_e\, dA$ for thin slice of area dA

volume of revolution: $\bar{x} = \frac{1}{V}\int_V x_e\, dV$ $\bar{y} = \frac{1}{V}\int_V y_e\, dV$ for disk of volume dV

where (x_e, y_e) are the coordinates of the centre-of-mass of the slice/disk

Moments of Inertia

thin, uniform plate: $I = \int_V r^2 dm = \rho t \int_A r^2 dA$ for area dA a distance r from the axis

volume of revolution: $I = \int_V r^2 dm = \rho \int_V r^2 dV$ for shell with volume dV at r from axis

radius of gyration: $R = \sqrt{\frac{I}{m}}$ where $m = \rho V$ (for thin plate, $m = \rho A t$)

Arc length of a curve $s = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

Surface area of a volume of revolution $A = 2\pi \int_a^b y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

Average value of a function $y_{av} = \frac{1}{b-a}\int_a^b y\, dx$