Math 187: Methods of Integration – Name that Method!

Name the method(s) which should be used to solve each integration problem. Methods include:

- basic substitution
- using a trig identity
- using an inverse trig form
- integration by parts
- trig substitution

When appropriate, also specify what *u* you would use. (Don't bother to complete the full integration.)

1. $\int \frac{x^5 dx}{x^6+9}$

2. $\int \left(5x^4-2\right)^{12} x^3 dx$

3. $\int \frac{x dx}{\sqrt{1-x^2}}$

4. $\int \frac{dx}{\sqrt{1-x^2}}$

5. $\int \frac{\sin\theta \, d\theta}{\cos\theta+1}$

6. $\int \left(\sin x+\cos x\right)^2 dx$

7. $\int \frac{e^{3x}}{1+e^{3x}} dx$

8. $\int e^{-x} \cos 4x \, dx$

9. $\int \frac{\ln y}{y} dy$

10. $\int \sin^2 x \cos^2 x \, dx$

11. $\int \sqrt{4+x^2} \, dx$

12. $\int \frac{\sin\sqrt{x}}{\sqrt{x}} dx$

13. $\int \frac{dx}{x^2+6x+25}$

14. $\int e^{\sin^2 x} \sin x \cos x \, dx$

15. $\int \cos^2 x \tan^2 x \, dx$

16. $\int \tan^2 x \sec^2 x \, dx$

17. $\int \tan^3 x \sec^3 x \, dx$

18. $\int x^2 \cos x \, dx$

19. $\int \cot x \sec^2 x \, dx$

20. $\int \frac{dx}{\left(x^2-4\right)^{3/2}}$

21. $\int \tan^{-1} x \, dx$

22. $\int x\sqrt{x-1} \, dx$

Answers:

1. $\int \frac{x^5 dx}{x^6+9}$ – basic substitution with $u = x^6$

2. $\int (5x^4-2)^{12} x^3 dx$ – basic substitution with $u = 5x^4 - 2$

3. $\int \frac{x dx}{\sqrt{1-x^2}}$ – basic substitution with $u = 1 - x^2$ (or you could use trig substitution with $\sin u = x$ if you insist – ouch!)

4. $\int \frac{dx}{\sqrt{1-x^2}}$ – this is just the $\sin^{-1} x$ form with a = 1 (or you could use trig substitution with $\sin u = x$ if you insist)

5. $\int \frac{\sin\theta \, d\theta}{\cos\theta + 1}$ – basic substitution with either $u = \cos\theta$ or $u = \cos\theta + 1$

6. $\int (\sin x + \cos x)^2 \, dx$ – expand out, change $\sin^2 x + \cos^2 x$ to 1, then substitute with either $u = \sin x$ or $u = \cos x$

7. $\int \frac{e^{3x}}{1+e^{3x}} dx$ – basic substitution with $u = e^{3x}$

8. $\int e^{-x} \cos 4x \, dx$ – integration by parts (can use either $u = e^{-x}$ and $dv = \cos 4x \, dx$ or the other way around with $u = \cos 4x$ and $dv = e^{-x} \, dx$)

9. $\int \frac{\ln y}{y} dy$ – basic substitution with $u = \ln y$

10. $\int \sin^2 x \cos^2 x \, dx$ – use power reducing formulae for both squared terms, multiply out, will get one term in $\cos^2 2x$, so use power reducing formula again

11. $\int \sqrt{4+x^2} \, dx$ – trig substitution with $x = 2\tan\theta$

12. $\int \frac{\sin\sqrt{x}}{\sqrt{x}} dx$ – basic substitution with $u = \sqrt{x}$

13. $\int \frac{dx}{x^2+6x+25}$ – complete the square in the denominator, will get $\tan^{-1} x$ form

14. $\int e^{\sin^2 x} \sin x \cos x \, dx$ – basic substitution with $u = \sin^2 x$

15. $\int \cos^2 x \tan^2 x \, dx$ – replace tan with sin/cos, cosines will cancel, then replace $\sin^2 x$ term by power-reducing formula

16. $\int \tan^2 x \sec^2 x \, dx$ – basic substitution with $u = \tan x$

17. $\int \tan^3 x \sec^3 x \, dx$ – change $\tan^2 x$ to secants using the Pythagorean identity, then pair the remaining tan with a secant to get $u = \sec x$ and $du = \sec x \tan x \, dx$

18. $\int x^2 \cos x \, dx$ – integration by parts with $u = x^2$ and $dv = \cos x \, dx$

19. $\int \cot x \sec^2 x \, dx$ – change the cot to 1/tan, then use basic substitution with $u = \tan x$ and $du = \sec^2 x$

20. $\int \frac{dx}{(x^2-4)^{3/2}}$ – trig substitution with $x = 2\sec\theta$

21. $\int \tan^{-1} x \, dx$ – integration by parts with $u = \tan^{-1} x$ and $dv = dx$

22. $\int x\sqrt{x-1} \, dx$ – either integration by parts with $u = x$ and $dv = \sqrt{x-1} \, dx$, or (sneaky!) basic substitution with $u = x - 1$ to get $\int (u+1)\sqrt{u} \, du$