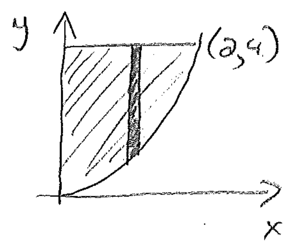
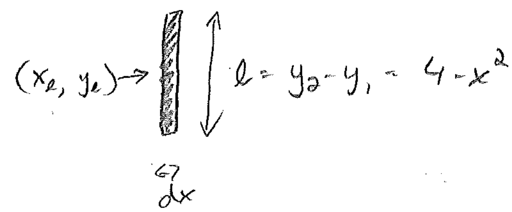
Math 187 –Tutorial on Centroids and MOI

1. Consider the region bounded by the curve $y = x^2$ and the lines $x = 0$ and $y = 4$. We will be calculating the coordinates of the centre-of-mass using the steps below.

   a) Sketch the region below.





   b) Find dA for a vertical slice.

$$dA = l\,dx = (4 - x^2)\,dx$$

   c) Find (x_e, y_e) in terms of x.

$$x_e = x$$

$$y_e = \frac{y_1 + y_2}{2} = \frac{x^2 + 4}{2}$$

   d) Set up (but don't bother to evaluate) an integral to calculate the area A of this region.

$$A = \int_A dA = \int_0^2 (4 - x^2)\,dx$$

   e) Set up (but don't bother to evaluate) an integral to calculate $\bar{x}$.

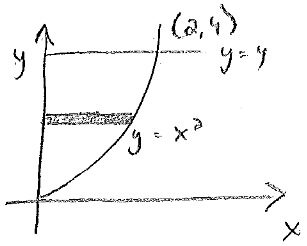
$$\bar{x} = \frac{1}{A}\int_A x_e\,dA = \frac{1}{A}\int_0^2 x(4 - x^2)\,dx$$

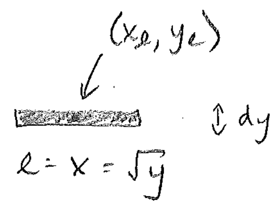
   f) Set up (but don't bother to evaluate) an integral to calculate $\bar{y}$.

$$\bar{y} = \frac{1}{A}\int_A y_e\,dA = \frac{1}{A}\int_0^2 \frac{(4 + x^2)}{2}(4 - x^2)\,dx$$

2. Consider the region bounded by the curve $y = x^2$ and the lines $x = 0$ and $y = 4$. (Yes, it's exactly the same region as for question 1, but with different slices.)

a) Sketch the region below.





b) Find dA for a horizontal slice.

$$dA = l\,dy = \sqrt{y}\,dy$$

c) Find (x_e, y_e) in terms of y.

$$x_e = \frac{x_2 + x_1}{2} = \frac{\sqrt{y}}{2}$$

$$y_e = y$$

d) Set up (but don't bother to evaluate) an integral to calculate the area A of this region.

$$A = \int_A dA = \int_0^4 \sqrt{y}\,dy$$

e) Set up (but don't bother to evaluate) an integral to calculate $\bar{x}$.

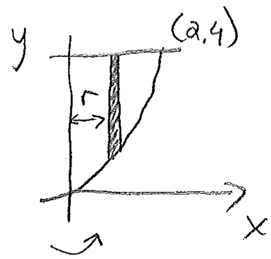
$$\bar{x} = \frac{1}{A}\int_A x_e\,dA = \frac{1}{A}\int_0^4 \frac{\sqrt{y}}{2}\cdot\sqrt{y}\,dy$$

f) Set up (but don't bother to evaluate) an integral to calculate $\bar{y}$.

$$\bar{y} = \frac{1}{A}\int_A y_e\,dA = \frac{1}{A}\int_0^4 y\sqrt{y}\,dy$$

3. Consider the region bounded by the curve $y = x^2$ and the lines $x = 0$ and $y = 4$. We will be calculating the moment of inertia about the y-axis using the steps below.

a) Sketch the region below.



$r = x$

b) Find dA for a slice that is parallel to the axis of rotation.

$$dA = (4 - x^2)\,dx$$ from before

c) Find r in terms of the variable you will be integrating with respect to.

$r = x$

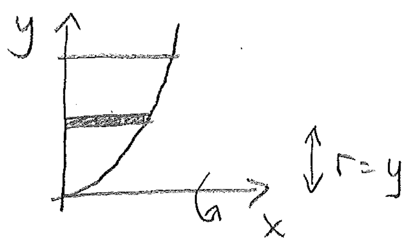
d) Set up (but don't bother to evaluate) an integral to calculate the moment of inertia I of this region.

$$I = \rho t \int_A r^2\,dA$$

$$= \rho t \int_0^2 x^2 (4 - x^2)\,dx$$

4. Consider the region bounded by the curve $y = x^2$ and the lines $x = 0$ and $y = 4$. We will be calculating the moment of inertia about the x-axis using the steps below.

a) Sketch the region below.



b) Find dA for a slice that is parallel to the axis of rotation.

$$dA = \sqrt{y}\, dy$$ from before

c) Find r in terms of the variable you will be integrating with respect to.

$$r = y$$

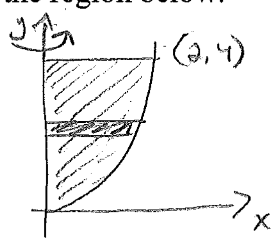
d) Set up (but don't bother to evaluate) an integral to calculate the moment of inertia I of this region.

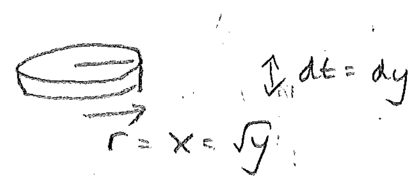
$$I = \rho t \int_A r^2 \, dA$$

$$= \rho t \int_0^4 y^2 \sqrt{y}\, dy$$

5. Consider the solid of revolution created by rotating about the y-axis the region bounded by the curve $y = x^2$ and the lines $x = 0$ and $y = 4$. We will be calculating the coordinates of the centre-of-mass using the steps below.

a) Sketch the region below.





b) Find dV for a disk.

$$dV = \pi r^2 dt$$
$$= \pi (\sqrt{y})^2 dy$$
$$= \pi y\, dy$$

c) Find (x_e, y_e) in terms of the variable of integration.

$x_e = 0$ by symmetry

$y_e = y$

d) Set up (but don't bother to evaluate) an integral to calculate the volume V of this region.

$$V = \int_V dV = \int_0^4 \pi y\, dy$$

e) One of the coordinates of the centroid is zero. Which one is it and why?

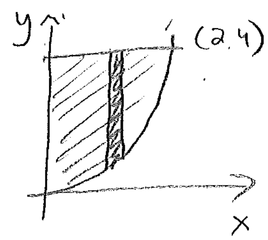
$\bar{x}$ is zero by symmetry

f) Set up (but don't bother to evaluate) an integral to calculate the other (non-zero) coordinate for the centroid.

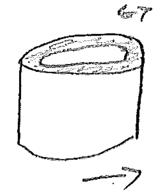
$$y_e = \frac{1}{V} \int_A y_e\, dV = \frac{1}{V} \int_0^4 y \cdot \pi y\, dy$$
$$= \frac{1}{V} \int_0^4 \pi y^2 dy$$

6. Consider the solid of revolution created by rotating about the y-axis the region bounded by the curve $y = x^2$ and the lines $x = 0$ and $y = 4$. We will be calculating the moment of inertia about the ~~x~~ y-axis using the steps below.

   a) Sketch the region below.



$dt = dx$



$h = y_2 - y_1 = 4 - x^2$

$r = x$

   b) Find dV for a shell.

$$dV = 2\pi r h\, dt = 2\pi\, x\,(4 - x^2)\, dx$$

   c) Find r in terms of the variable you will be integrating with respect to.

$$r = x$$

   d) Set up (but don't bother to evaluate) an integral to calculate the moment of inertia I of this region.

$$I = \rho \int_V r^2\, dV$$

$$= \rho \int_0^2 x^2 \cdot 2\pi x\,(4 - x^2)\,dx$$

$$= \rho \int_0^2 2\pi\, x^3\,(4 - x^2)\,dx$$