Section 25.2: Cont'd

Thursday, January 03, 2013
10:29 AM

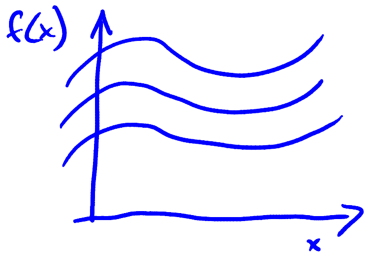
the indefinite integral:

$$\int f(x)\,dx = F(x) + C$$

geometric interpretation of the indefinite integral:

curve $F(x)$ has slope $f(x)$

so $F(x)$ is essentially the family of curves all with the same slope $f(x)$



all curves have same slope f(x) and the shift up or down doesn't affect the slope

properties of integrals: (you've probably guessed these)

$$\int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$$

$$\int c\, f(x)\,dx = c \int f(x)\,dx$$

power rule:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

for $n \neq -1$

examples:

$$\int (2x^4 - 5x^3 + 7)\,dx = \frac{2x^5}{5} - \frac{5x^4}{4} + 7x + C$$

$$\int \left(\sqrt{u} - \frac{5}{u^4}\right) du = \int \left(u^{1/2} - 5u^{-4}\right) du$$

$$= \frac{2}{3}u^{3/2} + \frac{5}{3}u^{-3} + C$$

$$\int \frac{2}{\sqrt[3]{y}}\,dy = \int 2y^{-1/3}\,dy$$

$$= 3y^{2/3} + C$$

example:

Find the equation of the curve $y = f(x)$ that passes through the point $(2, 3)$ if

$$\frac{dy}{dx} = 2x - 1$$

integrate:

$$y = \int \frac{dy}{dx}\,dx$$

$$= \int (2x-1)\,dx$$

$$= x^2 - x + C$$

note that the point $(2, 3)$ allows us to note that the point $(2, 3)$ allows us to find C:

$$3 = 4 - 2 + C$$

$$C = 1$$

$$\boxed{y = x^2 - x + 1}$$

example:

Find the equation of the curve for which the second derivative is $4x+1$. The curve has a slope of 18 at the point $(3, 9)$.

so $y'' = 4x+1$ $\left(\text{or } \frac{d^2y}{dx^2}\right)$

$$y' = \int y'' \, dx$$

$$= \int (4x+1) \, dx$$

$$= 2x^2 + x + C$$

at $x = 3$, $y' = 18$

$$18 = 2 \cdot 3^2 + 3 + C$$

$$C = -3$$

so $y' = 2x^2 + x - 3$

$$y = \int y' \, dx$$

$$= \int (2x^2 + x - 3) \, dx$$

$$= \frac{2}{3}x^3 + \frac{x^2}{2} - 3x + C^*$$

but when $x = 3$, $y = 9$

$$9 = 18 + \frac{9}{2} - 9 + c^*$$

$$c^* = -\frac{9}{2}$$

$$\boxed{y = \frac{2x^3}{3} + \frac{x^2}{2} - 3x - \frac{9}{2}}$$