Section 25.2: Cont'd

Friday, January 04, 2013
10:31 AM

Assignment #1 due on

Tues, Jan 22

Quiz #1 on

Fri, Jan 25

Section 25.1 → 26.2 inclusive

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integration:

recall the chain rule:

find the derivative of

$$f(x) = (3x^7 + 2)^9$$

$$f'(x) = 9(3x^7+2)^8 \; 21x^6$$

$$= 189 \, x^6 (3x^7+2)^8$$

so, the antiderivative of $189\,x^6(3x^7+2)^8$ is just $(3x^7+2)^9$

how, then, do we integrate things like $189\,x^6(3x^7+2)^8$ in a straightforward manner?

substitution:

let's start by integrating

$$\int (x^2+3)^4 \cdot 2x \, dx$$

let $u = x^2+3$
$du = 2x\,dx$

$$= \int u^4 \, du$$

recall:
$du = \frac{du}{dx} \cdot dx$

$$= \frac{u^5}{5} + C$$

$$= \frac{1}{5}(x^2+3)^5 + C$$

examples:

$$\int 4x^2 \sqrt{x^3+5} \, dx$$

let $u = x^3+5$
$du = 3x^2 dx$
$\frac{du}{3} = x^2 dx$

$$= \int 4 \, u^{1/2} \, \frac{du}{3}$$

$$= \frac{4}{3} \int u^{1/2} \, du$$

$$= \frac{4}{3} \cdot \frac{2}{3} u^{3/2} + C$$

$$= \frac{8}{9} (x^3+5)^{3/2} + C$$

$$\int (3x+2)^{10}\,dx$$

let $u = 3x+2$

$du = 3dx$

$\frac{du}{3} = dx$

$$= \int \frac{u^{10}\,du}{3}$$

$$= \frac{u^{11}}{3\cdot 11} + C$$

$$= \frac{(3x+2)^{11}}{33} + C$$

$$\int \frac{x\,dx}{(2+5x^2)^6}$$

let $u = 2+5x^2$

$du = 10x\,dx$

$\frac{du}{10} = x\,dx$

$$= \int u^{-6}\,\frac{du}{10}$$

$$= \frac{u^{-5}}{10(-5)} + C$$

$$= -\frac{(2+5x^2)^{-5}}{50} + C$$

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example:

Find the equation of the curve $y = f(x)$ which passes through the point $(2,-1)$ and has slope $\sqrt{6x-3}$.

$$\frac{dy}{dx} = \sqrt{6x-3}$$

$$y = \int \frac{dy}{dx}\, dx$$

$$= \int \sqrt{6x-3}\, dx$$

let $u = 6x-3$

$du = 6dx$

$$= \int u^{1/2} \frac{du}{6}$$

$$= \frac{2}{3}\frac{u^{3/2}}{6} + C$$

$$= \frac{1}{9}(6x-3)^{3/2} + C$$

point $(2,-1)$ is on curve

$$-1 = \frac{1}{9}(\ 9\ )^{3/2} + C$$

$$-1 = 3 + C$$

$$C = -4$$

$$\boxed{y = \frac{1}{9}(6x-3)^{3/2} - 4}$$