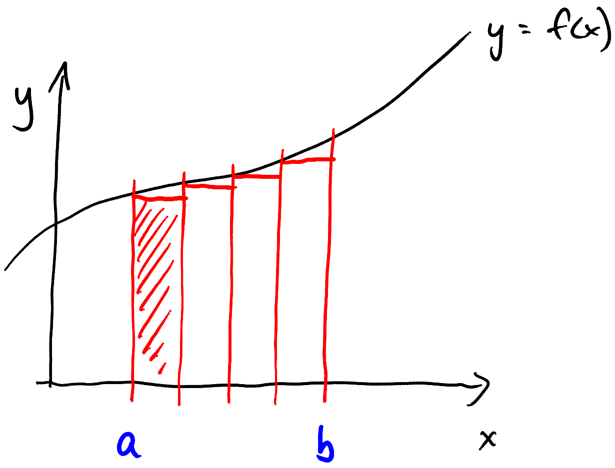
Section 25.3: cont'd

Monday, January 07, 2013
10:26 AM



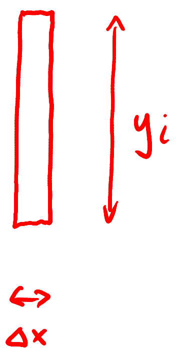
want the area under the curve from x = a to x = b

area ≈ sum of these rectangles

→ to make this estimate more accurate, just use more rectangles by decreasing the width of each rectangle

what is the area of each rectangle?

→ for simplicity, let each have the same width Δx



where y_i is the y-coordinate for that rectangle

so area $= \sum_{i=1}^{n} y_i \Delta x$

where n is the number

of rectangles

and Δx is the width

$$\text{area} = \sum_{i=1}^{n} f(x_i)\,\Delta x$$

x_i = the x-coordinate of the rectangle

calculus idea: we take the limit as $\Delta x \to 0$

$$\text{area} = \lim_{\Delta x \to 0} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

and we rewrite this now as

$$\text{area} = \int_a^b f(x)\,dx$$

"the integral from a to b of $f(x)\,dx$"

and it turns out that

$$\boxed{\int_a^b f(x)\,dx = F(b) - F(a)}$$

$F(b)$ is the antiderivative of $f(x)$ evaluated at b

example:

$$\int_0^2 (2x^3 + 3)\,dx = \left(\frac{2x^4}{4} + 3x\right)\Bigg|_0^2$$

$$= \left(\frac{2^4}{2} + 3\cdot 2\right) - 0$$

$$= 14$$