Section 25.4: cont'd

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10:32 AM

what about substitution for definite integrals?

consider

$$\int_1^2 6(x^3+1)^4 x^2 dx$$

method #1:

let $u = x^3+1$
$du = 3x^2 dx$
$\frac{du}{3} = x^2 dx$

$$= \int_{x=1}^{x=2} 6u^4 \frac{du}{3}$$

$$= \int_{x=1}^{x=2} 2u^4 du$$

$$= \frac{2}{5}u^5 \Big|_{x=1}^{x=2}$$

$$= \frac{2}{5}(x^3+1)^5 \Big|_{x=1}^{x=2}$$

$$= \frac{2}{5}(9)^5 - \frac{2}{5}(2)^5$$

$$= 23606.8$$

method #2

let $u = x^3+1$

$du = 3x^2\,dx$

$2\,du = 6x^2\,dx$

$$= \int_2^9 2u^4\,du$$

$$= \frac{2}{5}u^5 \Big|_2^9 = \frac{2}{5}\left(9^5 - 2^5\right) = \text{same answer}$$

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examples: (give answers to two decimals)

$$\int_0^1 \sqrt{5x+4}\,dx$$

let $u = 5x+4$

$du = 5\,dx$

$$= \int_{x=0}^{x=1} \frac{u^{1/2}\,du}{5}$$

$$= \frac{2}{3}\,\frac{1}{5}\,u^{3/2} \Big|_{x=0}^{x=1}$$

$$= \frac{2}{15}(5x+4)^{3/2} \Big|_{x=0}^{x=1}$$

$$= \frac{2}{15}\left[9^{3/2} - 4^{3/2}\right]$$

$$= \frac{38}{15} \approx 2.53$$

$(2.5\overline{3}$, really$)$

$$\int_2^3 \frac{(x^2+1)\,dx}{(x^3+3x)^2}$$

$du = \frac{du}{dx}\,dx$

let $u = x^3 + 3x$

$du = (3x^2 + 3)\,dx$

$\frac{du}{3} = (x^2 + 1)\,dx$

$$= \int_{14}^{36} \frac{du}{3}\, u^{-2}$$

when $x = 2$

$u = 2^3 + 3 \cdot 2$

$= 14$

$x = 3$

$u = 3^3 + 3 \cdot 3$

$= 36$

$$= -\frac{u^{-1}}{3}\Big|_{14}^{36}$$

$$= -\frac{1}{3 \cdot 36} + \frac{1}{3 \cdot 14}$$

$$= \frac{11}{756} \approx 0.01455$$

$$\approx 0.01$$

two useful properties to know about the definite integral:

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

[why?

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

$$= -\left[F(a) - F(b)\right]$$

$$= -\int_b^a f(x)\,dx \;]$$

→ if you switch the limits, the integral changes sign

$$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$$

example: (give exact answer)

$$\int_0^2 x\sqrt{2x^2+1}\,dx$$

let $u = 2x^2+1$
$du = 4x\,dx$
$\frac{du}{4} = x\,dx$

$$= \int_1^9 \frac{u^{1/2}\,du}{4}$$

when $x=0$, $u=1$
$x=2$, $u=9$

$$= \frac{2}{3}\cdot\frac{1}{4}u^{3/2}\Big|_1^9$$

$$= \frac{1}{6}\left(9^{3/2} - 1^{3/2}\right)$$

$$= \frac{26}{6} = \boxed{\frac{13}{3} = 4.\overline{3}}$$