# Section 25.5: Numerical Integration: The Trapezoid Rule

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10:29 AM

suppose you wish to find the area under a curve between $x = a$ and $x = b$

$$\text{Area} = \int_a^b f(x)\,dx$$

but what if you cannot compute this integral directly?

example: $\int_1^2 e^{-x^2}\,dx$

try substitution:

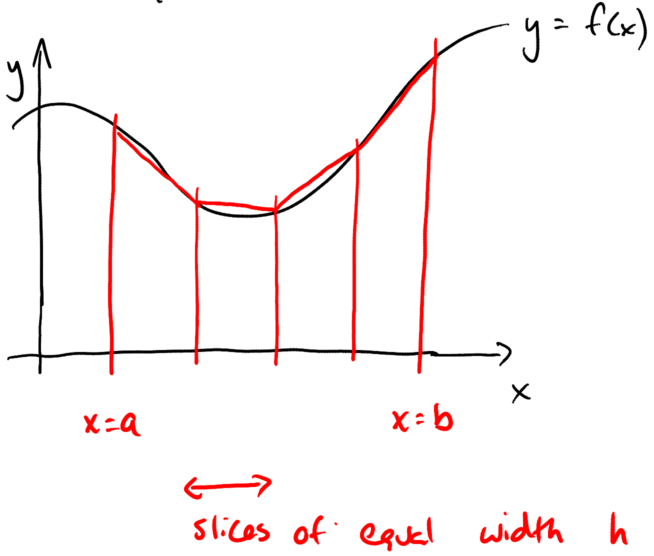
let $u = -x^2$

$du = -2x\,dx$

no x term in integral!

substitution has failed

numerical techniques



→ then we add the areas of these trapezoids

$$\int_a^b f(x)\,dx \approx \frac{h}{2}\left(y_0 + 2y_1 + 2y_2 + \ldots 2y_{n-1} + y_n\right)$$

example:

compute $\int_1^2 \frac{1}{x}\,dx$ by

a) using the trapezoidal rule with 5 steps
$n=5$

b) using integration

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trapezoidal rule:

width : $h = \dfrac{\text{upper limit} - \text{lower limit}}{\#\text{ steps}}$

$= \dfrac{2-1}{5} = \dfrac{1}{5}$

| $x$ | $y = \frac{1}{x}$ |
|---|---|
| 1 | 1 |
| 1.2 = 6/5 | 5/6 |
| 1.4 = 7/5 | 5/7 |
| 1.6 = 8/5 | 5/8 |
| 1.8 = 9/5 | 5/9 |
| 2 | 1/2 |

$$\int_1^2 \frac{1}{x}\,dx \approx \frac{h}{2}\left(y_0 + 2y_1 + 2y_2 + 2y_3 + 2y_4 + y_5\right)$$

$$\approx \frac{1}{\ldots}\cdot\frac{1}{\ldots}\left[1 + 2\left(\frac{5}{\ldots}\right) + 2\left(\frac{5}{\ldots}\right) + 2\left(\frac{5}{\ldots}\right) + 2\left(\frac{5}{\ldots}\right) + \frac{1}{\ldots}\right]$$

$$\approx \frac{1}{2}\cdot\frac{1}{5}\left[1 + 2\left(\frac{5}{6}\right) + 2\left(\frac{5}{7}\right) + 2\left(\frac{5}{8}\right) + 2\left(\frac{5}{9}\right) + \frac{1}{2}\right]$$

$$\approx \frac{1753}{2520}$$

$$\approx \underbrace{0.695635}_{\text{ridiculous number of sig figs for only 5 steps}}$$

exact answer:

$$\int_1^2 \frac{1}{x}\,dx = \ln x \Big|_1^2$$

$$= \ln 2 - \cancelto{0}{\ln 1}$$

$$= \ln 2$$

$$\approx 0.693147$$

So the trapezoidal rule is accurate to within 3 parts in 700
→ out by 0.36%

trapezoidal rule works <u>really</u> well for smooth, boring curves that don't change rapidly

there is one situation where numerical integration is the <u>prefered</u> technique

Suppose you wish to find $\int_a^b f(x)\,dx$ but you don't actually have an expression for $f(x)$, just a bunch of data points

example:

| $x$ | $y = f(x)$ |
|---|---|
| 20 | 12 |
| 21 | 13 |
| 22 | 11 |
| 23 | 9 |
| 24 | 8 |

$h = 1$ step size

just plug values into the trapezoidal rule directly

note: to use the trapezoidal rule, the x-values need equal spacing so that $h$ is a constant

$$\int_{20}^{24} f(x)\,dx = \frac{h}{2}\left(y_0 + 2y_1 + 2y_2 + 2y_3 + y_4\right)$$

$$= \frac{1}{2}\left(12 + 2\cdot 13 + 2\cdot 11 + 2\cdot 9 + 8\right)$$

$$= 43$$