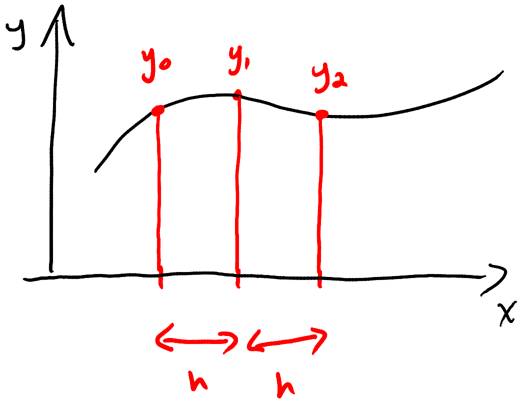
Section 25.6: Simpson's Rule

Thursday, January 10, 2013
10:28 AM

Instead of trapezoids, what could we do to improve the accuracy of our numerical integration?

- fit with a <u>curve</u> instead of a straight line



← idea is to fit these three points with a quadratic

$y = ax^2 + bx + c$

then, the area of this region is just

$$\frac{h}{3}\left(y_0 + 4y_1 + y_2\right)$$

details in handout if you are interested

so, start stacking:

one region with quadratic

$$\int_a^b f(x)\,dx \approx A_1 + A_2 + A_3 + A_4 + \dots A_n$$

$$\approx \frac{h}{3}\left(y_0 + 4y_1 + y_2\right) + \frac{h}{3}\left(y_2 + 4y_3 + y_4\right) + \dots$$

$$\approx \frac{h}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \dots 4y_{n-1} + y_n\right)$$

note: $h = \dfrac{b-a}{n}$

width

n = number of steps

and n must be $\underline{\underline{\text{even}}}$

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example:

Estimate the value of

$$\int_0^2 \sqrt{x^3+1}\,dx$$

with 4 steps using

a) trapezoidal rule
b) Simpson's rule

Round your answer to two decimal places.

x	$y = \sqrt{x^3+1}$
0	1
0.5	1.0607
1	1.4142
1.5	2.0917
2	3

trapezoid:

$$\int_a^b f(x)\,dx \approx \frac{h}{2}\left(y_0 + 2y_1 + 2y_2 + 2y_3 + y_4\right)$$

$$\approx 3.2833$$

$$\approx 3.28$$

Simpson's

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + y_4\right)$$
$$\approx 3.2396$$
$$\approx 3.24$$

If you use the Ti-89, this integral is 3.24131

trapezoid value is 1.3% off
Simpson's is 0.052% off