Section 26.1: cont'd

Friday, January 11, 2013
10:33 AM

Example:

A skier, starting from rest, skis down a slope with an acceleration (in m/s^2) given by

$$a = \frac{600t}{(60+0.5t^2)^2}$$

where t is the time in seconds. Find the magnitude of the skier's velocity as a function of time.

$$v = \int a\,dt$$

$$= \int \frac{600t\,dt}{(60+0.5t^2)^2}$$

let $u = 60+0.5t^2$

$du = t\,dt$

$$= \int 600u^{-2}\,du$$

$$= 600\,\frac{u^{-1}}{-1} + C$$

$$= -\frac{600}{u} + C$$

$$= -\frac{600}{60+0.5t^2} + C$$

at $t=0$, $v=0$

$$0 = -\frac{600}{60} + C$$

$$C = 10$$

$$\boxed{v(t) = -\frac{600}{60+0.5t^2} + 10}$$

Once sugar is dumped into a coffee mug, it dissolves with a time rate of change in the mass of $\frac{10}{\sqrt[3]{t+1}}$ (in g/s). Find the mass m of the sugar dissolved as a function of time. If 30g were initially dumped into the coffee, how long does it take for all of the sugar to dissolve?

$$\frac{dm}{dt} = \frac{10}{\sqrt[3]{t+1}}$$

$$m = \int \frac{dm}{dt}\, dt$$

$$= \int \frac{10}{\sqrt[3]{t+1}}\, dt$$

$$= \frac{3}{2} 10 (t+1)^{2/3} + C$$

$$m = 15(t+1)^{2/3} + c$$

$$m = 15(t+1)^{2/3} + C$$

at $t=0$, $m=0$

$$0 = 15 + C$$
$$C = -15$$

$$\boxed{m = 15(t+1)^{2/3} - 15}$$

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how long to dissolve all of it?

at this time, $m = 30$

$$m = 15(t+1)^{2/3} - 15$$
$$30 = 15(t+1)^{2/3} - 15$$
$$45 = 15(t+1)^{2/3}$$
$$3 = (t+1)^{2/3}$$
$$3^3 = (t+1)^2$$
$$\sqrt{27} = t+1$$
$$t = \sqrt{27} - 1 \approx 4.2 \text{ seconds}$$