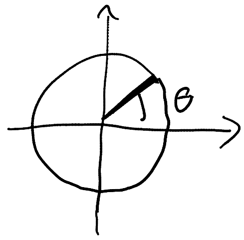
Section 26.1: Cont'd

Monday, January 14, 2013
10:29 AM

example:

Consider a rotating object.



$$\theta = \theta(t)$$

$$\omega = \frac{d\theta}{dt} \qquad \text{(angular velocity)}$$

$$\alpha = \frac{d\omega}{dt} \qquad \text{(angular acceleration)}$$

You are given the angular acceleration of a helicopter blade:

$$\alpha = \sqrt{8t+1}$$

Find θ if $\omega = 0$ and $\theta = 0$ when $t = 0$.

$$\alpha = \sqrt{8t+1}$$

$$\omega = \int \alpha \, dt$$

$$= \int \sqrt{8t+1} \, dt$$

let $u = 8t+1$

$du = 8\,dt$

$\frac{du}{8} = dt$

$$= \int \frac{u^{1/2}\,du}{8}$$

$$= \frac{1}{8}\,\frac{2}{3}\,u^{3/2} + C_1$$

$$= \frac{1}{12}(8t+1)^{3/2} + C_1$$

at $t=0$, $\omega=0$ so $0 = \frac{1}{12} 1^{3/2} + C_1$

$$C_1 = -\frac{1}{12}$$

$$\theta = \int \omega \, dt$$

$$= \int \left(\frac{1}{12} (8t+1)^{3/2} - \frac{1}{12} \right) dt$$

let $u = 8t+1$

$du = 8\,dt$

$\frac{du}{8} = dt$

$$= \int \left(\frac{1}{12} u^{3/2} - \frac{1}{12} \right) \frac{du}{8}$$

$$= \frac{1}{96} \frac{2}{5} u^{5/2} - \frac{1}{96} u + C_2$$

$$= \frac{1}{240} (8t+1)^{5/2} - \frac{1}{96} (8t+1) + C_2$$

at $t=0$, $\theta=0$ so

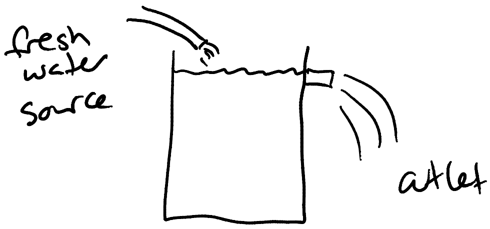
$$0 = \frac{1}{240} - \frac{1}{96} + C_2$$

$$= -\frac{1}{160} + C_2 \qquad \therefore C_2 = +\frac{1}{160}$$

$$\theta = \frac{1}{240} (8t+1)^{5/2} - \frac{1}{96} (8t+1) + \frac{1}{160}$$

$$\boxed{\theta = \frac{1}{240} (8t+1)^{5/2} - \frac{t}{12} - \frac{1}{240}}$$

Consider a tank full of brine (saltwater mixture) with a hose coming in which is adding fresh water to the tank and an outlet which is draining the mixture.



The mass m of salt in the tank has a rate of change of

$$\frac{dm}{dt} = -\frac{t}{\sqrt[3]{t^2+1000}} \quad \text{in } g/min$$

At time $t=0$, $m=100$. How long does it take for m to be zero?

$$m = \int \frac{dm}{dt}\, dt$$

$$= \int \frac{-t}{\sqrt[3]{t^2+1000}}\, dt$$

let $u = t^2+1000$
$du = 2t\, dt$
$\frac{du}{2} = t\, dt$

$$= \int -\frac{u^{-1/3}\, du}{2}$$

$$= -\frac{1}{2}\ \frac{3}{2}\ u^{2/3} + C$$

$$m = -\frac{3}{4}\left(t^2+1000\right)^{2/3} + C$$

at $t=0,\ m=100$ $\rightarrow$ find C

$$100 = -\frac{3}{4}\left(1000\right)^{2/3} + C$$

$$175 = C$$

$$m = -\frac{3}{4}\left(t^2+1000\right)^{2/3} + 175$$

Solve for t when $m=0$

$$0 = -\frac{3}{4}\left(t^2+1000\right)^{2/3} + 175$$

$$+\frac{3}{4}\left(t^2+1000\right)^{2/3} = 175$$

$$\left(t^2+1000\right)^{2/3} = \frac{700}{3}$$

$$t^2+1000 = \left(\frac{700}{3}\right)^{3/2}$$

$$t^2 = \left(\frac{700}{3}\right)^{3/2} - 1000$$

$$t \approx \pm\ 50.6382$$

$$\approx 51 \text{ min}$$