Section 26.2: Areas by Integration

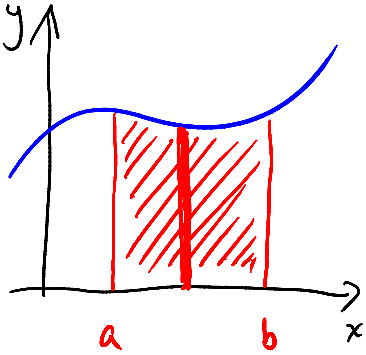
Tuesday, January 15, 2013
10:28 AM

recall: area under a curve:

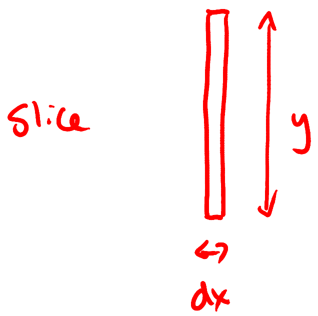
$$\int_a^b f(x)\,dx$$

in this section, we'll learn how to set up the integrals and then evaluate them

the big idea:



→ carve into little slices, then sum with an integral



area of the slice

$$dA = y\,dx$$

and

the sum of

$$A = \int_A dA$$

total area

each of the little areas

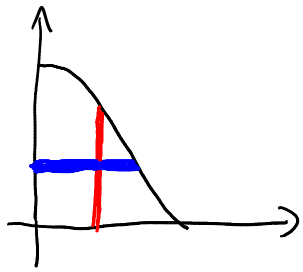
replacement for limits

↳ we are summing over this area

$$A = \int_A y \, dx$$

↑ substitute for y and integrate

but, consider:



Can do either:

$dA = y\,dx$ - vertical slice

or

$dA = x\,dy$ - horizontal slice

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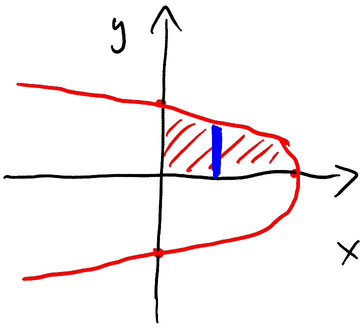
example:

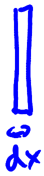
Find the area in the first quadrant bounded by $x = 4 - y^2$.



note: when $x = 0$, $y = \pm 2$
$y = 0$, $x = 4$

$x = 4 - y^2$
$y^2 = 4 - x$
$y = \pm\sqrt{4 - x}$
↑ top curve is

method #1: vertical slices



$\updownarrow\ y = \sqrt{4-x}$

$y = \sqrt{4-x}$

$dA = y\,dx = \sqrt{4-x}\,dx$

$$A = \int_A dA$$

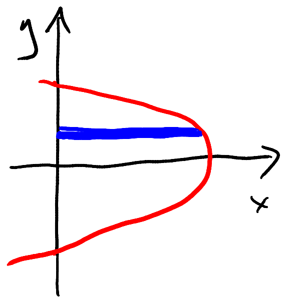
$$= \int_0^4 \sqrt{4-x}\,dx$$

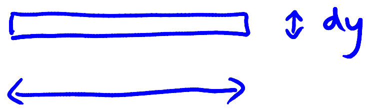
$$= -\frac{2}{3}(4-x)^{3/2}\Big|_0^4$$

$$= 0 - \left(-\frac{2}{3}4^{3/2}\right)$$

$$= \frac{16}{3}$$

method #2: horizontal slices





$x = 4 - y^2$

$$dA = x\,dy$$
$$= (4-y^2)\,dy$$

$$A = \int_A dA$$

$$= \int_0^2 (4-y^2)\,dy$$

$$= \left(4y - \frac{y^3}{3}\right)\Big|_0^2$$

$$= 8 - 8/3$$

$$= \frac{16}{3}$$ $\leftarrow$ same as before