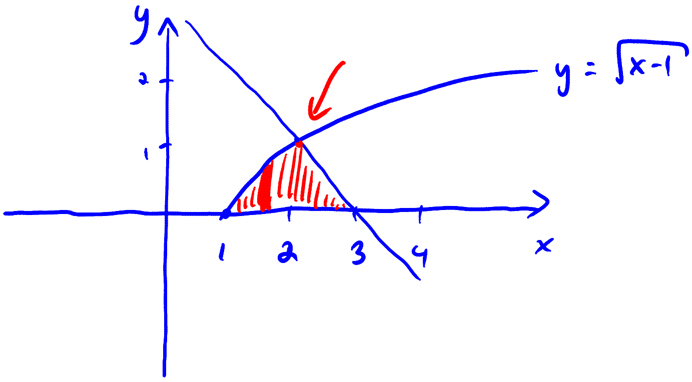
Section 26.2: cont'd

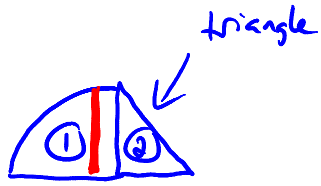
Wednesday, January 16, 2013
10:28 AM

example:

Find the area bounded by $y = \sqrt{x-1}$, $y = 3 - x$, and $y = 0$.



method #1 : vertical slices



area 1

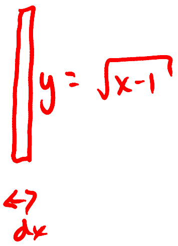
$$A_1 = \int_{A_1} dA$$

$$= \int_1^2 \sqrt{x-1}\,dx$$

$$= \frac{2}{3}(x-1)^{3/2}\Big|_1^2$$

$$= \frac{2}{3} - 0$$

$$= \frac{2}{3}$$



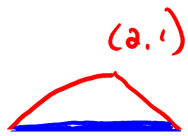
$dA = y\,dx = \sqrt{x-1}\,dx$

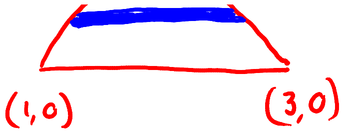
A_2 = area of triangle
$= \frac{1}{2}bh = \frac{1}{2}$

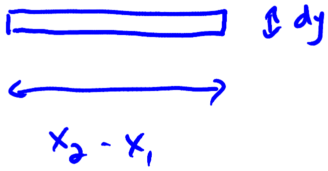
$$A_{total} = A_1 + A_2 = 2/3 + 1/2 = 7/6$$

---

method #2:







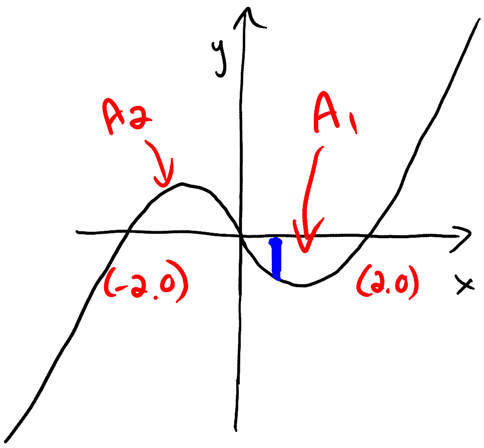
$$y_1 = \sqrt{x_1 - 1}$$
$$y_1^2 = x_1 - 1$$
$$x_1 = y_1^2 + 1$$

$$y_2 = 3 - x_2$$
$$x_2 = 3 - y_2$$

$$dA = (x_2 - x_1)\,dy$$
$$= \left[3 - y - (y^2 + 1)\right] dy$$
$$= (2 - y - y^2)\,dy$$

$$A = \int_A dA$$
$$= \int_0^1 (2 - y - y^2)\,dy$$
$$= \left[2y - \frac{y^2}{2} - \frac{y^3}{3}\right]\Bigg|_0^1$$
$$= 2 - \frac{1}{2} - \frac{1}{3} - 0$$
$$= \frac{7}{6}$$

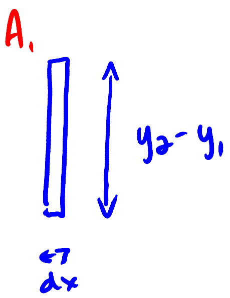
Find the area bounded by $y = x^3 - 4x$ and the x-axis.



x-intercepts:
$$y = x(x^2 - 4)$$
$$= x(x-2)(x+2)$$

$$x = 0, \pm 2$$

by symmetry $A_1 = A_2$



$$dA_1 = (y_2 - y_1)\,dx$$
$$= [0 - (x^3 - 4x)]\,dx$$
$$= (4x - x^3)\,dx$$

$$A_1 = \int_0^2 (4x - x^3)\,dx$$

$$= \left(2x^2 - \frac{x^4}{4}\right)\Big|_0^2$$

$$= 8 - 4 = 4$$

$$A = 2 \cdot A_1 = 8$$

one last point: if you are not careful and just do

$$A = \int_{-2}^{2} (x^3 - 4x)\,dx$$
$$= \text{zero} \qquad (!?!)$$

because the two "areas" will cancel

---

example: Find the area bounded by the lines

$x = 4$

$y = -3$

$y = 3$

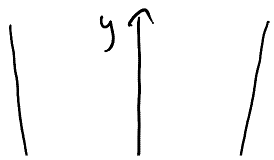
and the curve $y = x^2 - 9$ as shown in the diagram. Round your answer to one decimal point.

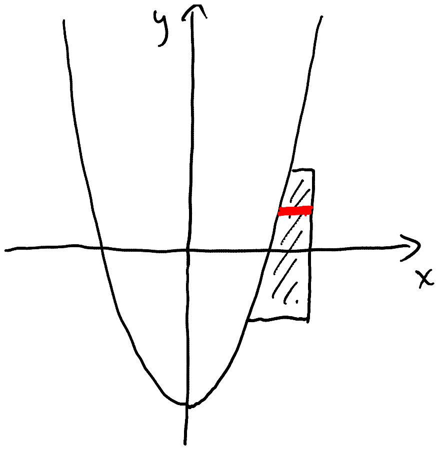
$y = x^2 - 9$

$y + 9 = x^2$

$x = \sqrt{y+9}$

horizontal slices

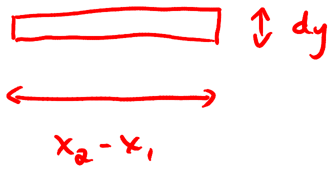




horizontal slices

$y + 9 = x^2$

$x = \sqrt{y+9}$



$$dA = (x_2 - x_1)\,dy$$
$$= \left(4 - \sqrt{y+9}\right) dy$$

$$A = \int_A dA$$
$$= \int_{-3}^{3} \left(4 - \sqrt{y+9}\right) dy$$
$$= \left[4y - \frac{2}{3}(y+9)^{3/2}\right] \Bigg|_{-3}^{3}$$
$$= \left[12 - \frac{2}{3}(12)^{3/2}\right] - \left[-12 - \frac{2}{3}(6)^{3/2}\right]$$
$$= 6.08515$$
$$= 6.1$$