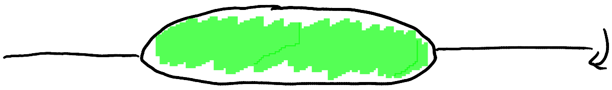
Section 26.3: Volumes by Integration

Thursday, January 17, 2013
10:27 AM

in this section, we will learn methods to find volumes of objects with rotational symmetry

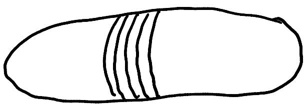
→ at the end of this course, we'll address arbitrary volumes

consider a cucumber:



nice symmetry about this axis

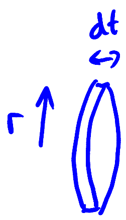
big idea: carve this up into a number of slices



slices

and the total volume is then just the sum of the individual slices:

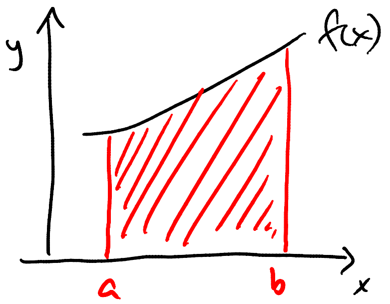
$$V = \int_V dV$$



each slice is a disk whose volume is

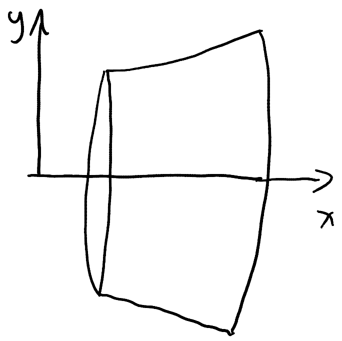
$$dV = \pi r^2 dt$$

Consider a function $f(x)$ in a region $a \le x \le b$. Consider also the shaded area between the curve and the x-axis.



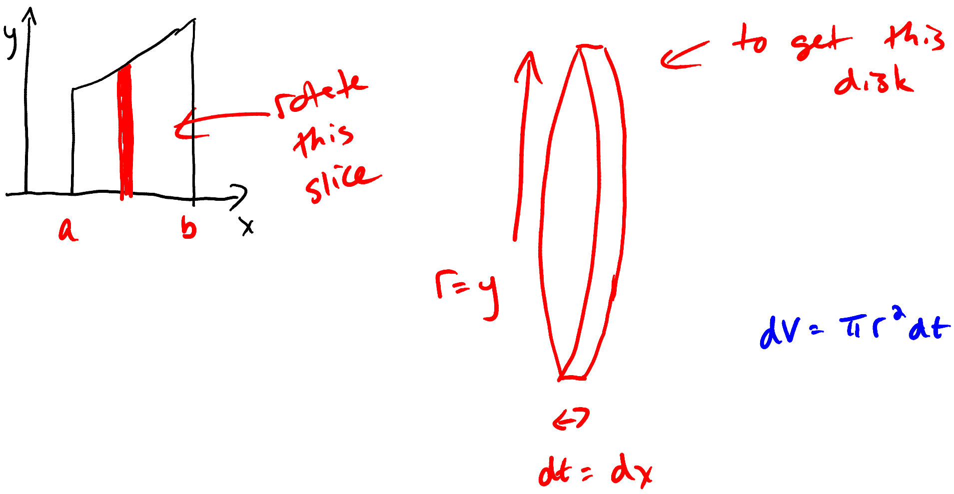
if we then rotate this about the x-axis, we'll generate

a "solid of revolution"



it's like a "filled-in" lampshade

how to find the volume of this figure?



$dV = \pi r^2 dt$

$$V = \int_V \pi r^2 \, dt$$

$$= \int_a^b \pi y^2 \, dx \quad \Leftarrow \text{ and } y = f(x)$$

$\rightarrow$ this is called the "disk method"

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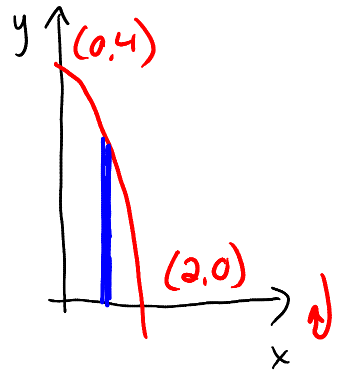
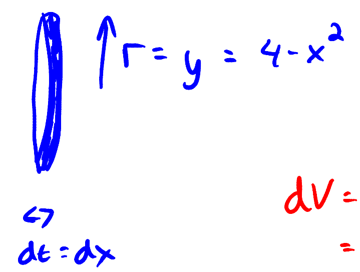
example: Calculate the volume of the solid of revolution generated by rotating the following area around the x-axis. The area is that region under the curve $y = 4 - x^2$ in the first quadrant.





$$dV = \pi r^2 \, dt$$
$$= \pi (4 - x^2)^2 \, dx$$

$$V = \int_V dV$$

$$= \int_0^2 \pi (4 - x^2)^2 \, dx$$

$$= \int_0^2 \pi (16 - 8x^2 + x^4) \, dx$$

$$= \pi \left(16x - \frac{8x^3}{3} + \frac{x^5}{5} \right) \Big|_0^2$$

$$= \pi \left(32 - \frac{64}{3} + \frac{32}{5} \right)$$

$$= \frac{256\pi}{15} \approx 53.6$$

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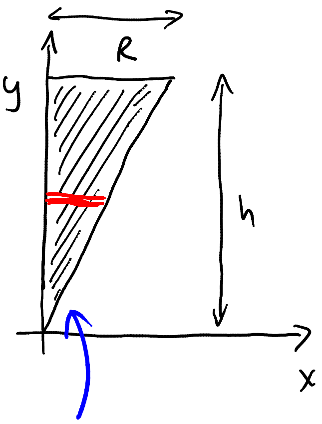
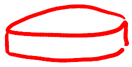
Consider the triangle in the following diagram which is then rotated about the y-axis. What is the volume of the resulting cone?





$\updownarrow dt = dy$

$r = x$

now write x in terms of y

what's the equation of the line?

$y = mx + b$

$y = \frac{h}{R} x$

so $x = \frac{Ry}{h}$

$$dV = \pi r^2 dt$$
$$= \pi x^2 dy$$
$$= \frac{\pi R^2}{h^2} y^2 dy$$

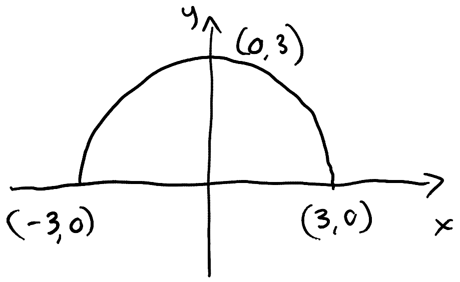
$$V = \int_V dV$$
$$= \int_0^h \frac{\pi R^2}{h^2} y^2 dy$$
$$= \frac{\pi R^2}{h^2} \frac{y^3}{3} \Big|_0^h$$
$$= \frac{\pi R^2 h^3}{3h^2} - 0$$

$$= \frac{1}{3} \pi R^2 h$$

example: calculate the volume of revolution for the curve

$$y = \sqrt{9 - x^2}$$

rotated around the x-axis.



note: the solid is a sphere with radius 3