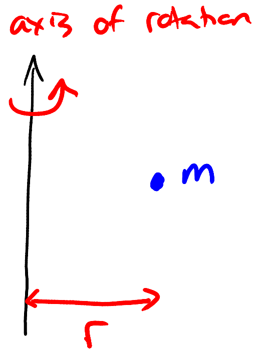
Section 26.5: Moments of Inertia

Wednesday, January 23, 2013
10:48 AM

moment of inertia of a point mass m at a distance r from an axis is



$I = mr^2$ about that axis

calculus idea:

so, for a small chunk of mass dm

$$dI = r^2\,dm$$

and, for an extended object

$$I = \int_M dI = \int_M r^2\,dm$$

special cases:

for an object of uniform density ρ
(recall that $dm = \rho\,dV$)

$$I = \int_V r^2 \rho\,dV$$

$$= \rho \int_V r^2\,dV$$

and for a thin flat plate of thickness t,
(recall that $dV = t\,dA$)

$$I = \rho t \int_A r^2\,dA$$

textbook calls this "k"

radius of gyration:

if an object of mass M_{TOT} and moment of inertia I were shrunk to a point, that point would be located at a distance

R_{gyr}

to have the same moment of inertia as the initial object

R_{gyr} = distance from the axis of rotation

so $I = M_{TOT} R_{gyr}^2$

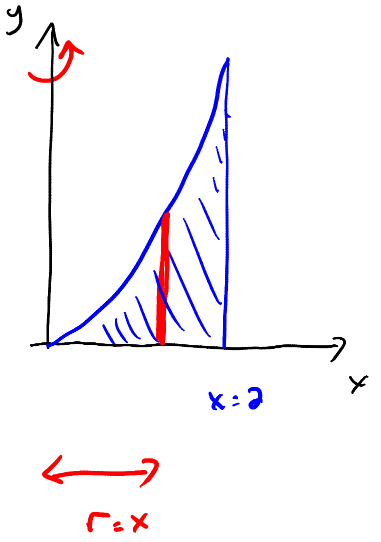
$$R_{gyr} = \sqrt{\frac{I}{M_{TOT}}}$$

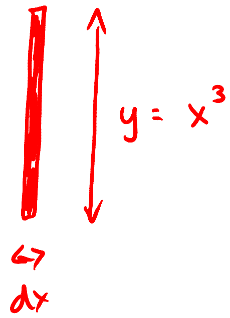
---

example:

Find the moment of inertia with respect to the y-axis of a thin plate of density ρ and thickness t which covers the region bounded by bounded by

$$y = x^3, \quad x = 2, \quad \text{and} \quad y = 0.$$





$$dA = y\,dx = x^3\,dx$$

$$I = \rho t \int_A r^2\,dA$$

$$= \rho t \int_0^2 x^2\, x^3\,dx$$

$$= \rho t \int_0^2 x^5\,dx$$

$$= \rho t \left. \frac{x^6}{6} \right|_0^2$$

$$= \frac{32}{3} \cdot \rho t$$