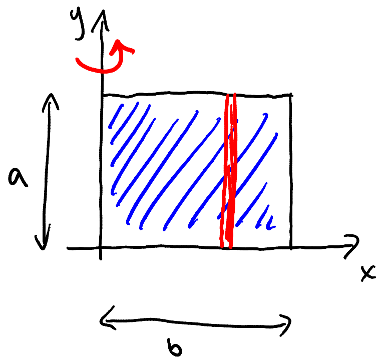
Section 26.5: cont'd

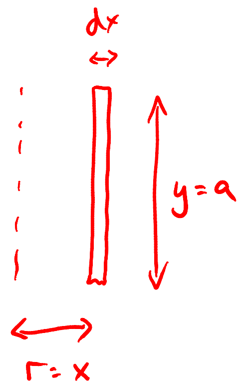
Monday, January 28, 2013
10:28 AM

example:

Find the moment of inertia of a rectangular plate of sides a & b with respect to side a. Express the result in terms of the mass of the plate. Assume that the plate has uniform density ρ and uniform thickness t.



note: for calculating I, easiest to choose slices (for thin plates) that are parallel to the axis



$dA = a\,dx$

for a thin plate:

$$I = \rho t \int_A r^2 \, dA$$

$$= \rho t \int_0^b x^2 a \, dx$$

note: a & b are constants

$$= \rho t a \frac{x^3}{3} \Big|_0^b$$

$$= \rho t a \frac{b^3}{3}$$

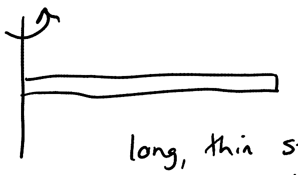
mass of plate:

$\Longleftarrow \quad m = \rho V = \rho abt$

$$= \frac{mb^2}{3}$$

note: doesn't depend on a!

important result:



long, thin stick of length ℓ, then the moment of inertia about that axis is $I = \frac{1}{3} m\ell^2$

So, if in your Mech classes, you use (for a thin plate) a slice perpendicular to the axis, then you need to adjust by a factor of $\frac{1}{3}$.

Similarly, shells are the best method of calculating I for solids of revolution, but if you have to use a disk then recall

$$I_{disk} = \frac{1}{2} m r^2$$

and so you need to adjust by a factor of $\frac{1}{2}$.

example:

Find the radius of gyration with respect to the y-axis for the solid of revolution made by rotating around the y-axis the region bounded by

$$y = x^3, \quad x = 0, \quad \text{and} \quad y = 8.$$

Round your answer to two decimals

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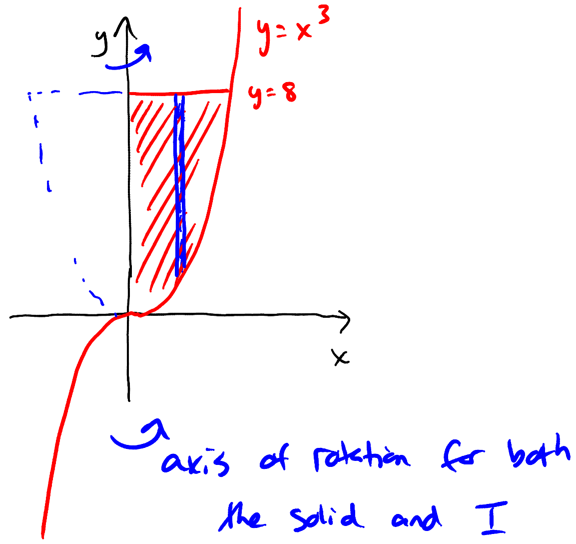

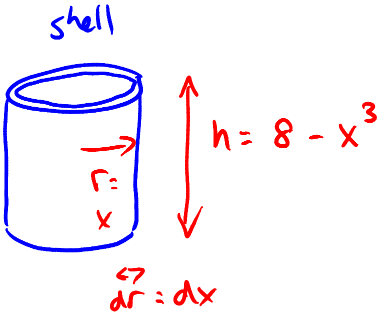

$$dV = 2\pi r h\, dr$$
$$= 2\pi x(8-x^3)\,dx$$

for a solid of revolution,

$$I = \rho \int_V r^2\, dV$$

$$= \rho \int_0^2 x^2\, 2\pi x (8-x^3)\, dx$$

$$= 2\pi\rho \int_0^2 (8x^3 - x^6)\, dx$$

$$= 2\pi\rho \left(\frac{8x^4}{4} - \frac{x^7}{7}\right)\Big|_0^2$$

$$= 2\pi\rho \left(32 - \frac{128}{7}\right)$$

$$= \frac{192\pi}{7}\rho \qquad \approx 27.4\,\pi\rho$$

need to find volume: but we already know dV

$$V = \int_V dV$$

$$= \int_0^2 2\pi x (8-x^3)\, dx$$

$$= \int_0^2 2\pi (8x - x^4)\, dx$$

$$= 2\pi \left(\frac{8x^2}{2} - \frac{x^5}{5} \right) \Big|_0^2$$

$$= 2\pi \left(16 - \frac{32}{5} \right)$$

$$= \frac{96\pi}{5} = 19.2\pi$$

so $M_{TOT} = \rho V$
$= \frac{96\pi\rho}{5}$

$$R_{gyr} = \sqrt{\frac{I}{M_{TOT}}}$$

$$= \sqrt{\frac{192\pi\rho}{7} \cdot \frac{5}{96\pi\rho}}$$

$$= \sqrt{\frac{10}{7}} \approx 1.19523$$

$$\approx 1.20$$ (to 2 decimals)

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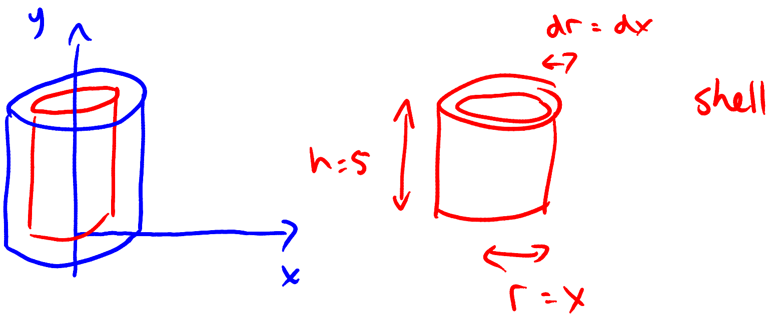
example: Find the moment of inertia for a cylinder of radius 3 and height 5 about its axis of symmetry.

You may leave your answer in terms of the density ρ (or, if you prefer, k).



$$dV = 2\pi r h\, dr$$
$$= 2\pi x\, 5\, dx$$

$$I = \rho \int_V r^2\, dV$$

$$= \rho \int_0^{3} x^2 \, 10\pi \, x \, dx$$

$$= 10\pi\rho \int_0^3 x^3 \, dx$$

$$= 10\pi\rho \left. \frac{x^4}{4} \right|_0^3$$

$$= 10\pi\rho \, \frac{81}{4}$$

$$= \frac{405}{2}\pi\rho \qquad = 202.5\,\pi\rho$$