Section 26.6: Other Applications

Tuesday, January 29, 2013
10:30 AM

we'll look at

- work done by a variable force
- average value of a function
- arclength
- area of a surface of revolution

} detailed in exercises

note: we will omit the force due to liquid pressure (p 789 in 9th ed)

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work done by a variable force: 1D

recall from physics: work = force · distance
= Fd
= Fd cos θ ← 2D
= $\vec{F} \cdot \vec{d}$ ↳ full definition

if, then, $F = F(x)$ ← the size of the force depends on where you are

then $dW = F(x)\,dx$

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$$W = \int_a^b F(x)\,dx$$

for a force exerted from $x=a$ to $x=b$

(note: assuming 1D)

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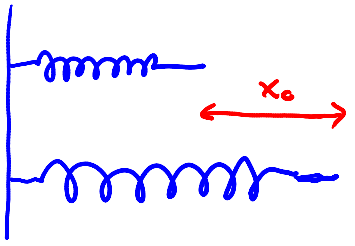
example: What is the work done by an external force in stretching a spring from the equilibrium position out to a distance x_0?



unstretched

stretched

$F_{ext} = kx$

$$W = \int_a^b F(x)\,dx$$

$$= \int_0^{x_0} kx\,dx$$

$$= \frac{kx^2}{2}\Big|_0^{x_0}$$

$$= \frac{1}{2}kx_0^2$$

← should look familiar!

Similar problems: what if direction were changing while the magnitude of F was constant?

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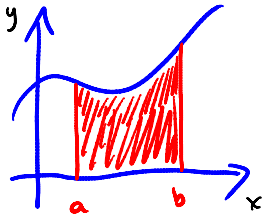
average value of a function:

$$y_{av} = \frac{\int_a^b y\,dx}{b-a}$$



← integral is the area under the curve

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example: Find the work done in winding up 75m of a 125m rope that has a mass of 12.5 kg. From that, also calculate the average force required.

let x = length of cable that has been wound up

then the length of cable left is just $(125\,m - x)$

and the mass of cable left is just $(125\,m - x)\left(\frac{12.5\,kg}{125\,m}\right)$

$= 0.100\,(125 - x)$

and the weight of cable left is just

$$0.100\,\frac{kg}{m}\,(125m - x)\left(9.8\,\frac{N}{kg}\right)$$

$\left(\frac{N}{m}\right)(125m - x)\left(1.0\ \frac{N}{kg}\right)$

$$W = \int_a^b F(x)\,dx$$

$$= \int_0^{75} 0.98\,(125 - x)\,dx$$

$$= 0.98\left(125x - \frac{x^2}{2}\right)\Big|_0^{75}$$

$$= 6431.25\ Nm \quad \leftarrow \text{book's answer}$$

$$= 6400\ J \quad \leftarrow \text{my preferred answer}$$
$$\text{or } 6.4\ kJ$$

↖ work done

average force:

$$y_{av} = \frac{\int_a^b y\,dx}{b-a}$$

which means

$$F_{av} = \frac{\int_a^b F(x)\,dx}{b-a}$$

we just found this

$$= \frac{6431\ Nm}{75\ m}$$

$$= 86\ N$$

example: A hemispherical tank of radius R is full of water. Find the work done in

pumping at the tank.