Section 28.2: The Basic Logarithm Form

Friday, February 01, 2013
10:30 AM

recall: $\frac{d}{dx}(\ln x) = \frac{1}{x}$

where the domain of $\ln x$ is $x > 0$

values of x for which the function $\ln x$ is defined

so $\int \frac{1}{x}\,dx = \ln|x| + C$

this ensures that the logarithm is defined

examples:

① $\int \frac{dx}{x-3} = \ln|x-3| + C$

② $\int \tan\theta \, d\theta$

$= \int \frac{\sin\theta \, d\theta}{\cos\theta}$ let $u = \cos\theta$, $du = -\sin\theta\, d\theta$

$= \int -\frac{du}{u}$

$= -\ln|u| + C$

$= -\ln|\cos\theta| + C$ ← perfectly acceptable answer #1

$= \ln|\sec\theta| + C$ ← perfectly acceptable answer #2

note: similarly,

$$\int \cot\theta\, d\theta = \ln|\sin\theta| + C \qquad \text{(textbook)}$$

$$= -\ln|\csc\theta| + C \qquad \text{(Gilles' materials)}$$

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③ $$\int \frac{x^3\,dx}{1-x^4}$$

let $u = 1-x^4$
$du = -4x^3 dx$

$$= \int -\frac{du}{4u}$$

$$= -\frac{1}{4}\ln|u| + C$$

$$= -\frac{1}{4}\ln|1-x^4| + C$$

④ $$\int \frac{e^{-3x}}{2+5e^{-3x}}\,dx$$

let $u = 2+5e^{-3x}$
$du = -15e^{-3x}dx$

$$= \int \frac{du}{-15u}$$

$$= -\frac{1}{15}\ln|u| + C$$

$$= -\frac{1}{15}\ln|2+5e^{-3x}| + C$$ ⇐ perfectly acceptable answer #1

$$= -\frac{1}{15}\ln\underbrace{(2+5e^{-3x})}_{\text{because}} + C$$ ⇐ perfectly acceptable answer #2

because $2+5e^{-3x}$ is always positive

and, unfortunately, you can rewrite this expression:

$$= -\frac{1}{15} \ln\left(\frac{2e^{3x}+5}{e^{3x}}\right) + C$$

$$= -\frac{1}{15}\left[\ln(2e^{3x}+5) - \ln e^{3x}\right] + C$$

$$= -\frac{1}{15}\left[\ln(2e^{3x}+5) - 3x\right] + C$$

$$= \frac{1}{5}x - \frac{1}{15}\ln(2e^{3x}+5) + C$$

perfectly acceptable answer #3

(5) $\displaystyle\int \frac{\sin 2\theta}{1-\cos^2\theta}\, d\theta$

$$= \int \frac{2\sin\theta\cos\theta\, d\theta}{1-\cos^2\theta}$$

let $u = 1-\cos^2\theta$

$du = -2\cos\theta(-\sin\theta)\,d\theta$

$= 2\sin\theta\cos\theta\,d\theta$

$$= \int \frac{du}{u}$$

$$= \ln|u| + C$$

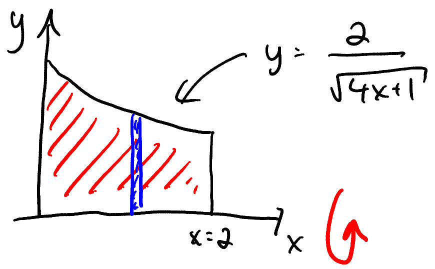
$$= \ln\left|1-\cos^2\theta\right| + C$$

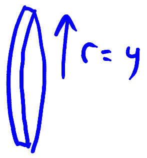
either

$$= 2 \ln|\sin\theta| + C$$ either

where might we find such integrals?

example: Find the volume of the solid of revolution created by rotating the following area about the x-axis.





$dt = dx$

$$\begin{aligned} dV &= \pi r^2 \, dt \\ &= \pi y^2 \, dx \\ &= \pi \frac{4}{4x+1} \, dx \end{aligned}$$

$$\begin{aligned} V = \int dV &= \int_0^2 \frac{4\pi}{4x+1} \, dx \\ &= \frac{4\pi}{4} \ln|4x+1| \Big|_0^2 \\ &= \pi \ln 9 - \pi \cancel{\ln 1}^{\,0} \\ &= \pi \ln 9 \end{aligned}$$

or $2\pi \ln 3$