Section 28.3: The Exponential Form

Monday, February 04, 2013
10:29 AM

recall:

$$\frac{d}{dx}(e^x) = e^x$$

therefore $$\int e^x\,dx = e^x + C$$

---

examples:

(1) $\int e^{3x}\,dx$

let $u = 3x$
$du = 3dx$

$= \int \frac{1}{3} e^u\,du$

$= \frac{1}{3} e^u + C$

$= \frac{1}{3} e^{3x} + C$

in general:

$$\int e^{ax}\,dx = \frac{e^{ax}}{a} + C \quad \text{for } a \neq 0$$

(2) $\int x e^{x^2}\,dx$

let $u = x^2$
$du = 2x\,dx$

$= \int e^u \frac{du}{2}$

$= \frac{1}{2} e^u + C$

$= \frac{1}{2} e^{x^2} + C$

$= \frac{1}{2} e^{x^2} + C$

note: $\int_0^1 x e^{x^2} dx = \frac{1}{2} e^{x^2} \Big|_0^1 = \frac{1}{2}(e - 1)$

③ $\int \cos 2\theta \; e^{\sin\theta \cos\theta} \, d\theta$

$= \int \cos 2\theta \; e^{\frac{1}{2}\sin 2\theta} \, d\theta$

let $u = \frac{1}{2}\sin 2\theta$

$du = \cos 2\theta \, d\theta$

$= \int e^u \, du$

$= e^{\frac{1}{2}\sin 2\theta} + C$

④ $\int_2^3 \sqrt{e^{3y} - 3e^{2y}} \, dy$ (round to 2 decimals)

$= \int_2^3 \sqrt{e^{2y}(e^y - 3)} \, dy$

$= \int_2^3 e^y \sqrt{e^y - 3} \, dy$ $\qquad \sqrt{e^{2y}} = (e^{2y})^{\frac{1}{2}}$

let $u = e^y - 3$

$du = e^y \, dy$

$= \int_{y=2}^{y=3} u^{\frac{1}{2}} \, du$

$$= \frac{2}{3} u^{3/2} \Big|_{y=2}^{y=3}$$

$$= \frac{2}{3} (e^y - 3)^{3/2} \Big|_2^3$$

$$= \frac{2}{3} (e^3 - 3)^{3/2} - \frac{2}{3} (e^2 - 3)^{3/2}$$

$$\approx 40.9516$$

$$\approx 40.95$$

$(e^x)^2 = e^{2x}$

⑤ $\int (e^x + e^{-x})^2 \, dx$

$$= \int (e^{2x} + 2 + e^{-2x}) \, dx$$

$$= \frac{e^{2x}}{2} + 2x - \frac{e^{-2x}}{2} + C$$

⑥ $\int \frac{4 \, d\theta}{\sec\theta \, e^{\sin\theta}}$

$$= \int 4 \cos\theta \, e^{-\sin\theta} \, d\theta$$

let $u = -\sin\theta$

$du = -\cos\theta \, d\theta$

$$= \int -4 e^u \, du$$

$$= -4 e^u + C$$

$$= -4 e^{-\sin\theta} + C$$

⑦ $\int_3^5 2 e^{R^2 + \ln R} \, dR$ (round to 2 sigfigs)

$$= \int_3^5 2 e^{R^2} e^{\ln R} \, dR \qquad \left(e^{\ln x} = x\right)$$

$$= \int_3^5 2 e^{R^2} R \, dR$$

let $u = R^2$

$du = 2R \, dR$

$$= \int_9^{25} e^u \, du$$

when $R = 3$, $u = 9$

$R = 5$, $u = 25$

$$= e^u \Big|_9^{25}$$

$$= e^{25} - e^9$$

$$\approx 7.20049 \times 10^{10}$$

$$\approx 7.2 \times 10^{10}$$