

## Section 28.4: Basic Trig Forms

Tuesday, February 05, 2013  
10:41 AM

from the derivatives of the six trig functions, we find that

$$\int \sin u \, du = -\cos u + C$$

$$\int \cos u \, du = \sin u + C$$

$$\int \sec^2 u \, du = \tan u + C$$

$$\int \csc^2 u \, du = -\cot u + C$$

$$\int \sec u \tan u \, du = \sec u + C$$

$$\int \csc u \cot u \, du = -\csc u + C$$

} formula sheet

examples:

$$\begin{aligned} \textcircled{1} \quad \int 2 \csc^2 5\theta \, d\theta &= \frac{2}{5} (-\cot 5\theta) + C \\ &= -\frac{2}{5} \cot 5\theta + C \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad \int e^x \sec e^x \tan e^x \, dx & \quad \left| \begin{array}{l} \text{let } u = e^x \\ du = e^x dx \end{array} \right. \\ &= \int \sec u \tan u \, du \\ &= \sec u + C \\ &= \sec e^x + C \end{aligned}$$

if the integral isn't in the form you want, try rewriting it, using trig identities, etc

$$\int \tan u \, du = \int \frac{\sin u}{\cos u} \, du \quad \begin{array}{l} \text{let } x = \cos u \\ dx = -\sin u \, du \end{array}$$

$$= \int -\frac{1}{x} \, dx$$

$$= -\ln |x| + C$$

$$= -\ln |\cos u| + C \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{ either}$$

$$= \ln |\sec u| + C$$

similarly,

$$\int \cot u \, du = \int \frac{\cos u}{\sin u} \, du$$

$$= \ln |\sin u| + C \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{ either}$$

$$= -\ln |\csc u| + C$$

what about  $\int \sec u \, du$  ?

need a trick:

$$\int \sec u \, du = \int \sec u \left( \frac{\sec u + \tan u}{\sec u + \tan u} \right) \, du$$

$$= \int \frac{\sec^2 u + \sec u \tan u}{\sec u + \tan u} \, du$$

$$\begin{array}{l} \text{let } x = \sec u + \tan u \\ dx = (\sec u \tan u + \sec^2 u) \, du \end{array}$$

$$= \int \frac{dx}{x}$$

$$= \ln |x| + C$$

$$= \ln |\sec u + \tan u| + C$$

similarly  $\int \csc u \, du = \ln |\csc u - \cot u| + C$

examples of where we'd use these:

$$\textcircled{1} \quad \int \cot \frac{\theta}{2} d\theta = 2 \ln \left| \sin \frac{\theta}{2} \right| + C$$

$$\begin{aligned} \textcircled{2} \quad \int \frac{3dx}{\cos 2x} &= \int 3 \sec 2x dx \\ &= \frac{3}{2} \ln \left| \sec 2x + \tan 2x \right| + C \end{aligned}$$

$$\textcircled{3} \quad \int \frac{1 + \sec^2 x}{x + \tan x} dx$$

$$\begin{aligned} \text{let } u &= x + \tan x \\ du &= (1 + \sec^2 x) dx \end{aligned}$$

$$\begin{aligned} &= \int \frac{du}{u} \\ &= \ln |u| + C \\ &= \ln |x + \tan x| + C \end{aligned}$$

$$\begin{aligned} \textcircled{4} \quad \int (1 + \sec x)^2 dx &= \int (1 + 2\sec x + \sec^2 x) dx \\ &= x + 2 \ln \left| \sec x + \tan x \right| \\ &\quad + \tan x + C \end{aligned}$$

$$\textcircled{5} \quad \int 5 \tan \theta \ln(\cos \theta) d\theta$$

$$\begin{aligned} \text{let } u &= \ln(\cos \theta) \\ du &= -\tan \theta d\theta \end{aligned}$$

$$= \int -5v \, dv$$

$$= -\frac{5}{2} v^2 + C$$

$$= -\frac{5}{2} [\ln(\cos \theta)]^2 + C$$