Section 28.4: Basic Trig Forms

Tuesday, February 05, 2013
10:41 AM

from the derivatives of the six trig functions, we find that

$$\int \sin u \, du = -\cos u + C$$

$$\int \cos u \, du = \sin u + C$$

$$\int \sec^2 u \, du = \tan u + C$$

$$\int \csc^2 u \, du = -\cot u + C$$

$$\int \sec u \tan u \, du = \sec u + C$$

$$\int \csc u \cot u \, du = -\csc u + C$$

formula sheet

examples:

① $\int 2\csc^2 5\theta \, d\theta$

$$= \frac{2}{5}\left(-\cot 5\theta\right) + C$$

$$= -\frac{2}{5}\cot 5\theta + C$$

② $\int e^x \sec e^x \tan e^x \, dx$

let $u = e^x$
$du = e^x \, dx$

$$= \int \sec u \tan u \, du$$

$$= \sec u + C$$

$$= \sec e^x + C$$

if the integral isn't in the form you want, try rewriting it, using trig identities, etc

$$\int \tan u \, du = \int \frac{\sin u}{\cos u} \, du$$

let $x = \cos u$
$dx = -\sin u \, du$

$$= \int -\frac{1}{x} \, dx$$

$$= -\ln|x| + C$$

$$= -\ln|\cos u| + C$$
$$= \ln|\sec u| + C$$
either

similarly,

$$\int \cot u \, du = \int \frac{\cos u}{\sin u} \, du$$

$$= \ln|\sin u| + C$$
$$= -\ln|\csc u| + C$$
either

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what about $\int \sec u \, du$?

need a trick:

$$\int \sec u \, du = \int \sec u \left(\frac{\sec u + \tan u}{\sec u + \tan u} \right) du$$

$$= \int \frac{\sec^2 u + \sec u \tan u}{\sec u + \tan u} \, du$$

let $x = \sec u + \tan u$
$dx = (\sec u \tan u + \sec^2 u) du$

$$= \int \frac{dx}{x}$$

$$= \ln|x| + C$$

$$= \ln|\sec u + \tan u| + C$$

similarly $\int \csc u \, du = \ln|\csc u - \cot u| + C$

examples of where we'd use these:

① $\int \cot \frac{\theta}{2}\, d\theta = 2 \ln \left| \sin \frac{\theta}{2} \right| + C$

② $\int \frac{3\,dx}{\cos 2x} = \int 3 \sec 2x\, dx$

$= \frac{3}{2} \ln \left| \sec 2x + \tan 2x \right| + C$

③ $\int \frac{1 + \sec^2 x}{x + \tan x}\, dx$

let $u = x + \tan x$
$du = (1 + \sec^2 x)\, dx$

$= \int \frac{du}{u}$

$= \ln |u| + C$

$= \ln \left| x + \tan x \right| + C$

④ $\int (1 + \sec x)^2\, dx$

$= \int (1 + 2\sec x + \sec^2 x)\, dx$

$= x + 2 \ln \left| \sec x + \tan x \right|$

$+ \tan x + C$

⑤ $\int 5 \tan \theta \ln(\cos \theta)\, d\theta$

let $u = \ln(\cos\theta)$
$du = -\tan \theta\, d\theta$

$$= \int -5 \, u \, du$$

$$= -\frac{5}{2} u^2 + C$$

$$= -\frac{5}{2} \left[\ln(\cos\theta) \right]^2 + C$$