

Section 28.4: cont'd

Wednesday, February 06, 2013
10:30 AM

$$\textcircled{6} \int \frac{\tan e^{-x}}{e^x} dx$$

$$= \int \textcircled{e^{-x}} \tan e^{-x} \textcircled{dx}$$

$$\text{let } u = e^{-x}$$

$$du = -e^{-x} dx$$

$$= - \int \tan u du$$

$$= + \ln |\cos u| + C$$

$$= \ln |\cos e^{-x}| + C$$

$$\textcircled{7} \int \frac{\tan 2x}{\cos 2x} dx$$

method #1:

$$= \int \frac{\textcircled{\sin 2x}}{\cos^2 2x} \textcircled{dx}$$

$$\text{let } u = \cos 2x$$

$$du = -2 \sin 2x dx$$

$$= \int -\frac{1}{2} \frac{du}{u^2}$$

$$= +\frac{1}{2} u^{-1} + C$$

$$= \frac{1}{2 \cos 2x} + C$$

} either

$$= \frac{1}{2} \sec 2x + C$$

method #2:

$$\begin{aligned} \int \frac{\tan 2x}{\cos 2x} dx &= \int \tan 2x \sec 2x dx \\ &= \frac{1}{2} \sec 2x + C \end{aligned}$$

$$\textcircled{8} \int \frac{1}{1 + \sin \theta} d\theta$$

hint: multiply by $\left(\frac{1 - \sin \theta}{1 - \sin \theta} \right)$

$$\begin{aligned} &= \int \frac{1}{1 + \sin \theta} \left(\frac{1 - \sin \theta}{1 - \sin \theta} \right) d\theta \\ &= \int \frac{1 - \sin \theta}{1 - \sin^2 \theta} d\theta \\ &= \int \frac{1 - \sin \theta}{\cos^2 \theta} d\theta \\ &= \int \left(\frac{1}{\cos^2 \theta} - \frac{\sin \theta}{\cos^2 \theta} \right) d\theta \\ &= \int \left(\sec^2 \theta - \frac{\sin \theta}{\cos^2 \theta} \right) d\theta \\ &= \tan \theta - \frac{1}{\cos \theta} + C \end{aligned}$$

$$\begin{aligned}
 &= \tan \theta - \frac{1}{\cos \theta} + C \\
 &= \tan \theta - \sec \theta + C \\
 &= \frac{\sin \theta - 1}{\cos \theta} + C
 \end{aligned}
 \left. \vphantom{\begin{aligned} &= \tan \theta - \frac{1}{\cos \theta} + C \\ &= \tan \theta - \sec \theta + C \\ &= \frac{\sin \theta - 1}{\cos \theta} + C \end{aligned}} \right\} \text{any of these}$$

⑨ $\int \sin x \cos x \, dx$

method #1:

let $u = \sin x$
 $du = \cos x \, dx$

$$\begin{aligned}
 &= \int u \, du \\
 &= \frac{u^2}{2} + C \\
 &= \frac{\sin^2 x}{2} + C
 \end{aligned}$$

method #2

let $u = \cos x$
 $du = -\sin x \, dx$

$$\begin{aligned}
 &= - \int u \, du \\
 &= - \frac{u^2}{2} + C \\
 &= - \frac{\cos^2 x}{2} + C
 \end{aligned}$$

method #3:

$$\begin{aligned}
 \int \sin x \cos x \, dx &= \int \frac{1}{2} \sin 2x \, dx \\
 &= - \frac{1}{4} \cos 2x + C
 \end{aligned}$$