Section 28.5: Other Trig Forms

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10:57 AM

What if we want to integrate $\int \sin^3 x \, dx$

or $\int \sin^2 x \, dx$?

need some trig identities:

Pythagorean:

$$\sin^2 x + \cos^2 x = 1$$
$$\tan^2 x + 1 = \sec^2 x$$
$$1 + \cot^2 x = \csc^2 x$$

power-reducing:

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$
$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

note: $\int \cos x \, dx = \sin x + C$

$$\int \cos(ax) \, dx = \frac{\sin(ax)}{a} + C$$

$\Leftarrow$ nice shortcut

$$\int \sin^2 x \, dx = \int \left(\frac{1 - \cos 2x}{2} \right) dx$$

note: even power → use power-reducing

$$= \int \left(\frac{1}{2} - \frac{1}{2} \cos 2x \right) dx$$

$$= \frac{1}{2} x - \frac{1}{4} \sin 2x + C$$

$$\int \sin^3 x \, dx = \int \sin^2 x \sin x \, dx$$

$$= \int (1-\cos^2 x) \sin x \, dx$$

$$= \int \sin x \, dx - \int \cos^2 x \sin x \, dx$$

let $u = \cos x$

$du = -\sin x \, dx$

$$= -\cos x + \int u^2 \, du$$

$$= -\cos x + \frac{u^3}{3} + C$$

$$= -\cos x + \frac{1}{3}\cos^3 x + C$$

note: use different strategies for $\sin^n x$:

when n is odd: use $(1-\cos^2 x)$ to replace powers of $\sin x$, leaving one extra factor of $\sin x$

when n is even: Use power-reducing formulas

example:

$$\int \sin^3 x \cos^6 x \, dx$$

$$= \int \sin x (1-\cos^2 x) \cos^6 x \, dx$$

$$= \int (\cos^6 x - \cos^8 x) \sin x \, dx$$

let $u = \cos x$

$du = -\sin x \, dx$

$$= \int (-u^6 + u^8) \, du$$

$$= -\frac{u^7}{7} + \frac{u^9}{9} + C$$

$$= \frac{-\cos^7 x}{7} + \frac{\cos^9 x}{9} + C$$