Section 28.5 cont'd:

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10:31 AM

more examples:

$$\int \cos^4\theta \, d\theta$$

$$= \int \left(\frac{1+\cos 2\theta}{2}\right)^2 d\theta$$

$$= \int \frac{1}{4}\left(1 + 2\cos 2\theta + \cos^2 2\theta\right) d\theta$$

$$= \int \frac{1}{4}\left(1 + 2\cos 2\theta + \frac{1+\cos 4\theta}{2}\right) d\theta$$

$$= \int \left(\frac{3}{8} + \frac{1}{2}\cos 2\theta + \frac{1}{8}\cos 4\theta\right) d\theta$$

$$= \frac{3}{8}\theta + \frac{\sin 2\theta}{4} + \frac{\sin 4\theta}{32} + C$$

integrals involving tan & sec: $\int \sec^n\theta \tan^m\theta \, d\theta$

if you let $u = \tan\theta$
$du = \sec^2\theta \, d\theta$

you let $u = \sec\theta$
$du = \sec\theta \tan\theta \, d\theta$

which to choose? Let's do some examples:

$$\int \tan^4\theta \boxed{\sec^2\theta \, d\theta}$$

let $u = \tan\theta$
$du = \sec^2\theta \, d\theta$

$$= \int u^4 du$$

$$= \frac{u^5}{5} + C$$

$$= \frac{u^5}{5} + C$$

$$= \frac{\tan^5\theta}{5} + C$$

$$\int \sec^4\theta \tan\theta \, d\theta$$

method #1:

let $u = \sec\theta$

$du = \sec\theta \tan\theta \, d\theta$

$$= \int \sec^3\theta \left[\sec\theta \tan\theta \, d\theta\right]$$

$$= \int u^3 \, du$$

$$= \frac{u^4}{4} + C$$

$$= \frac{\sec^4\theta}{4} + C$$

method #2:

$$\int \sec^4\theta \tan\theta \, d\theta =$$

$$= \int \sec^2\theta \tan\theta \boxed{\sec^2\theta \, d\theta}$$

$$= \int (1 + \tan^2\theta) \tan\theta \sec^2\theta \, d\theta$$

$$= \int (\tan\theta + \tan^3\theta) \sec^2\theta \, d\theta$$

let $u = \tan\theta$

$du = \sec^2\theta \, d\theta$

$$= \int (u + u^3) \, du$$

$$= \frac{u^2}{2} + \frac{u^4}{4} + C$$

$$= \frac{\tan^2\theta}{2} + \frac{\tan^4\theta}{4} + C$$

so if you have $\int \tan^{m}\theta \sec^{n}\theta \, d\theta$

if n is even, split into two:

eg. $\sec^6\theta = \sec^4\theta \underbrace{\sec^2\theta}$

this will be part of du

→ derivative of $\tan\theta$

replace with as many $(1+\tan^2\theta)$ as needed

if m is odd, split into:

$$\int \underbrace{\tan^{\square}\theta}_{\text{even power of tangents}} \sec^{\square}\theta \underbrace{\tan\theta\sec\theta\,d\theta}_{\text{derivative of secant}}$$

→ replace $\tan^2\theta$ with $(\sec^2\theta - 1)$ as many times as needed

if m is odd and n is even, can use either

if m is even and n odd → we won't ask you these!

identify strategies:

$\int \tan^2\theta \sec^{4}\theta \, d\theta$ power on secant is even

let $u = \tan\theta$
and use $\sec^2\theta = 1 + \tan^2\theta$

$$\int \tan^3\theta \sec^5\theta \, d\theta$$

power on tangent is odd

so let $u = \sec\theta$

$$\int \tan^3\theta \sec^4\theta \, d\theta$$

$\rightarrow$ use either

$$\int \tan^4\theta \, d\theta = \int \tan^2\theta(\sec^2\theta - 1)\, d\theta$$

$$= \int (\underbrace{\tan^2\theta \sec^2\theta}_{\text{great!}} - \tan^2\theta)\, d\theta$$

replace by $\sec^2\theta - 1$

note: $\int \sec^3\theta \, d\theta$ $\leftarrow$ awful! requires integration by parts (section 28.7)

note: integrals of cot and csc work <u>exactly</u> the same way

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example:

$$\int \sec^4 x \, dx$$

$$= \int (1 + \tan^2 x)\sec^2 x \, dx$$

let $u = \tan x$
$du = \sec^2 x \, dx$

$$= \int (1 + u^2)\, du$$

$$= u + \frac{u^3}{3} + C$$

$$= \tan x + \frac{\tan^3 x}{3} + C$$

$$\int \frac{\sin\theta \sec\theta}{\cot\theta} \, d\theta$$

$$= \int \frac{\sin\theta}{\cos\theta} \tan\theta \, d\theta$$

$$= \int \tan^2\theta \, d\theta$$

$$= \int (\sec^2\theta - 1)\, d\theta$$

$$= \tan\theta - \theta + C$$