Section 28.7: Integration by Parts
Tuesday, February 12, 2013
10:30 AM

consider $\int x^2 \ln x \, dx$

note: this integral is a product, and you cannot rewrite it to use basic substitution

(could substitute with $\int \frac{\ln x}{x} dx$, for example)

recall the product rule:

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

now integrate both sides wrt x:

$$\underbrace{\int \frac{d}{dx}(uv)\,dx}_{uv} = \int u \underbrace{\boxed{\frac{dv}{dx}\,dx}}_{dv} + \int v \underbrace{\boxed{\frac{du}{dx}\,dx}}_{du}$$

$$uv = \int u\,dv + \int v\,du$$

solve for $\int u\,dv$:

On Quiz #3 formula sheet

$$\boxed{\int u\,dv = uv - \int v\,du}$$

how does it work?

$$\int x^2 \ln x \, dx = \int \underbrace{\ln x}_{u} \; \underbrace{x^2 dx}_{dv}$$

let $u = \ln x$ $\qquad$ $v = \frac{1}{3}x^3$ $\qquad$ (ignore the +C for now)

$du = \frac{1}{x} dx$ $\qquad$ $dv = x^2 dx$

now write out the "parts" formula:

$$\int u \, dv = uv - \int v \, du$$

$$\int \ln x \; x^2 dx = \ln x \left(\tfrac{1}{3}x^3\right) - \underbrace{\int \tfrac{1}{3}x^3 \tfrac{1}{x} \, dx}$$

why is this cool? because we can integrate this

$$= \tfrac{1}{3}x^3 \ln x - \int \tfrac{1}{3} x^2 dx$$

$$= \tfrac{1}{3}x^3 \ln x - \tfrac{1}{9}x^3 + \underbrace{C}$$

each integral gives a constant, but we just lump them all together at the end

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example:

$$\int x \cos 2x \, dx$$

let $u = x$ $\qquad$ $v = \frac{1}{2}\sin 2x$

$du = dx$ $\qquad$ $dv = \cos 2x \, dx$

u dv

$$\int u\,dv = uv - \int v\,du$$

$$\int x\cos 2x\,dx = x\left(\tfrac{1}{2}\sin 2x\right) - \int \tfrac{1}{2}\sin 2x\,dx$$

$$= \tfrac{1}{2}x\sin 2x - \frac{1}{2}\cdot\left(-\frac{1}{2}\right)\cos 2x + C$$

$$= \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + C$$

guidelines for u & v:

idea: is to choose u so that du is simpler in form

and that dv is something you can integrate

two basic types

① $$\int \underbrace{x^n}_{u} \underbrace{\begin{Bmatrix} e^{ax} \\ \sin ax \\ \cos ax \end{Bmatrix} dx}_{dv}$$

notice that the dv's are cyclical in nature

$$② \int x^n \left\{ \begin{matrix} \ln x \\ \tan^{-1} x \\ \sin^{-1} x \end{matrix} \right\} dx$$

$$\underbrace{\left\{ \begin{matrix} \ln x \\ \tan^{-1} x \\ \sin^{-1} x \end{matrix} \right\}}_{u}$$

so $dv = x^n\,dx$

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example: $\displaystyle\int \frac{x\,dx}{\sqrt{2x-1}}$

let $u = x$ $\qquad v = \frac{2}{2}(2x-1)^{1/2}$

$du = dx$ $\qquad dv = (2x-1)^{-1/2}\,dx$

$$\int u\,dv = uv - \int v\,du$$

$$\int x(2x-1)^{-1/2}\,dx = x(2x-1)^{1/2} - \int (2x-1)^{1/2}\,dx$$

$$= x(2x-1)^{1/2} - \frac{2}{3}\frac{1}{2}(2x-1)^{3/2} + C$$

$$= x(2x-1)^{1/2} - \frac{1}{3}(2x-1)^{3/2} + C$$