Assignment #3 due on Tuesday, March 5

Quiz #3 on Friday, March 8

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what about $\int e^x \cos x \, dx$ ?

| D | I |
|---|---|
| $\cos x$ | $e^x$ |
| $-\sin x$ | $e^x$ |
| $-\cos x$ | $e^x$ |

$$\int e^x \cos x \, dx = e^x \cos x - \int (-\sin x) e^x \, dx$$

$$= e^x \cos x - (-\sin x) e^x + \int (-\cos x) e^x \, dx$$

$$\int e^x \cos x \, dx = e^x \cos x + e^x \sin x - \int e^x \cos x \, dx$$

$$+\int e^x \cos x \, dx \qquad\qquad + \int e^x \cos x \, dx$$

$$2 \int e^x \cos x \, dx = e^x \cos x + e^x \sin x$$

$$\int e^x \cos x \, dx = \frac{1}{2}\left(e^x \cos x + e^x \sin x\right) + C$$

non-tabular method for the same problem:

$$\int e^x \cos x \, dx$$

let $u = \cos x \qquad v = e^x$

$du = -\sin x \, dx \qquad dv = e^x dx$

$$= uv - \int v\,du$$

$$= e^x \cos x - \int e^x(-\sin x\,dx)$$

$$= e^x \cos x + \int e^x \sin x\,dx$$

let $u = \sin x$ $\quad$ $v = e^x$
$du = \cos x\,dx$ $\quad$ $dv = e^x\,dx$

$$= e^x \cos x + uv - \int v\,du$$

$$= e^x \cos x + e^x \sin x - \int e^x \cos x\,dx$$

$$2\int e^x \cos x\,dx = e^x \cos x + e^x \sin x$$

$$\int e^x \cos x\,dx = \tfrac{1}{2} e^x(\cos x + \sin x) + C$$

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Special cases:

$$\int \underbrace{\sin^{-1} x}_{u}\ \underbrace{dx}_{dv}$$

let $u = \sin^{-1} x$ $\quad$ $v = x$
$du = \frac{1}{\sqrt{1-x^2}}\,dx$ $\quad$ $dv = dx$

if $\sin^{-1} x\,dx$ were $dv$, we couldn't integrate it to get $v$ (not on formula sheet)

$$= uv - \int v\,du$$

$$= x\sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}}\,dx$$

let $f = 1 - x^2$
$df = -2x\,dx$

$$= x\sin^{-1} x - \int \left(\frac{-1}{2}\right) f^{-1/2}\,df$$

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$$= x\sin^{-1}x + \frac{1}{2}\,2\,t^{1/2} + C$$

$$= x\sin^{-1}x + \sqrt{1-x^2} + C$$

check: $\frac{d}{dx}\left( x\sin^{-1}x + \sqrt{1-x^2}\right)$

$$= x\cdot\frac{1}{\sqrt{1-x^2}} + \sin^{-1}x + \frac{1}{2}\frac{1}{\sqrt{1-x^2}}(-2x)$$

$$= \frac{x}{\sqrt{1-x^2}} + \sin^{-1}x - \frac{x}{\sqrt{1-x^2}}$$

$$= \sin^{-1}x$$