

Section 28.7: cont'd

Thursday, February 14, 2013  
10:31 AM

Assignment #3 due on  
Tuesday, March 5

Quiz #3 on Friday, March 8

What about  $\int e^x \cos x \, dx$  ?

D	I
$\cos x$	$e^x$
$-\sin x$	$e^x$
$-\cos x$	$e^x$

$$\int e^x \cos x \, dx = e^x \cos x - \int (-\sin x) e^x \, dx$$

$$= e^x \cos x - (-\sin x) e^x + \int (-\cos x) e^x \, dx$$

$$\int e^x \cos x \, dx = e^x \cos x + e^x \sin x + \int e^x \cos x \, dx$$

$$- \int e^x \cos x \, dx + \int e^x \cos x \, dx$$

$$2 \int e^x \cos x \, dx = e^x \cos x + e^x \sin x$$

$$\int e^x \cos x \, dx = \frac{1}{2} (e^x \cos x + e^x \sin x) + C$$

non-tabular method for the same problem:

$$\int e^x \cos x \, dx$$

$$\left[ \begin{array}{l} \text{let } u = \cos x \\ du = -\sin x \, dx \end{array} \right. \quad \left. \begin{array}{l} v = e^x \\ dv = e^x \, dx \end{array} \right.$$

$$\begin{aligned}
 &= uv - \int v du \\
 &= e^x \cos x - \int e^x (-\sin x dx) \\
 &= e^x \cos x + \int e^x \sin x dx
 \end{aligned}$$

$$\begin{array}{ll}
 \text{let } u = \sin x & v = e^x \\
 du = \cos x dx & dv = e^x dx
 \end{array}$$

$$\begin{aligned}
 &= e^x \cos x + uv - \int v du \\
 &= e^x \cos x + e^x \sin x - \int e^x \cos x dx
 \end{aligned}$$

$$2 \int e^x \cos x dx = e^x \cos x + e^x \sin x$$

$$\int e^x \cos x dx = \frac{1}{2} e^x (\cos x + \sin x) + C$$

Special cases:

$$\int \underbrace{\sin^{-1} x}_u dx \quad \underbrace{dx}_{dv}$$

$$\begin{array}{ll}
 \text{let } u = \sin^{-1} x & v = x \\
 du = \frac{1}{\sqrt{1-x^2}} dx & dv = dx
 \end{array}$$

if  $\sin^{-1} x dx$  were  $dv$ , we couldn't integrate it to get  $v$   
(look on formula sheet)

$$\begin{aligned}
 &= uv - \int v du \\
 &= x \sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}} dx
 \end{aligned}$$

$$\begin{array}{l}
 \text{let } f = 1-x^2 \\
 df = -2x dx
 \end{array}$$

$$= x \sin^{-1} x - \int \left(\frac{-1}{2}\right) f^{-\frac{1}{2}} df$$

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$$= x \sin^{-1} x + \frac{1}{2} 2 f^{\frac{1}{2}} + C$$

$$= x \sin^{-1} x + \sqrt{1-x^2} + C$$

check:  $\frac{d}{dx} (x \sin^{-1} x + \sqrt{1-x^2})$

$$= x \cdot \frac{1}{\sqrt{1-x^2}} + \sin^{-1} x + \frac{1}{2} \frac{1}{\sqrt{1-x^2}} (-2x)$$

$$= \frac{x}{\sqrt{1-x^2}} + \sin^{-1} x - \frac{x}{\sqrt{1-x^2}}$$

$$= \sin^{-1} x$$