# Section 28.8: Integration by Trig Substitution

Monday, February 18, 2013
10:49 AM

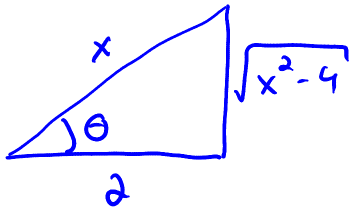
review, cont'd:

<u>example:</u>

if $x = 2\sec\theta$, find $\theta$, $\sin\theta$, and $\cot\theta$ in terms of $x$ (assume $x$ is positive)

$\sec\theta = \frac{x}{2}$

$\cos\theta = \frac{2}{x}$




$\theta = \cos^{-1}\left(\frac{2}{x}\right)$

$\sin\theta = \frac{\sqrt{x^2-4}}{x}$

$\cot\theta = \frac{2}{\sqrt{x^2-4}}$

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consider $\int \frac{1}{\sqrt{x^2+9}}\,dx$

← can't use basic substitution (no $x$ in numerator)

← neither $\arcsin\left(\frac{1}{\sqrt{9-x^2}}\right)$ or $\arctan\left(\frac{1}{x^2+9}\right)$

but what if $x^2+9$ were some nice perfect square that we could take the square root of ?

let $x = 3\tan\theta$

$dx = 3\sec^2\theta\, d\theta$

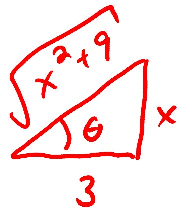
$$x^2+9 = 9\tan^2\theta + 9$$
$$= 9(\tan^2\theta + 1)$$
$$= 9\sec^2\theta$$

$$\sqrt{x^2+9} = \sqrt{9\sec^2\theta}$$
$$= 3\sec\theta$$

so $$\int \frac{dx}{\sqrt{x^2+9}} = \int \frac{3\sec^2\theta\, d\theta}{3\sec\theta}$$
$$= \int \sec\theta\, d\theta$$
$$= \ln|\sec\theta + \tan\theta| + C$$

$x = 3\tan\theta$

$\tan\theta = \frac{x}{3}$




$$\sec\theta = \frac{\sqrt{x^2+9}}{3}$$
$$\tan\theta = \frac{x}{3}$$

so $$\int \frac{dx}{\sqrt{x^2+9}} = \ln\left|\frac{\sqrt{x^2+9}}{3} + \frac{x}{3}\right| + C$$

wacky-looking, but true!

example:

$$\int \sqrt{4-x^2}\,dx$$

let $x = 2\sin\theta$
$dx = 2\cos\theta\,d\theta$

$$= \int \sqrt{4-4\sin^2\theta}\;2\cos\theta\,d\theta$$

$$= \int \sqrt{4(1-\sin^2\theta)}\;2\cos\theta\,d\theta$$

$$= \int \sqrt{4\cos^2\theta}\;2\cos\theta\,d\theta$$

$$= \int 2\cos\theta\;2\cos\theta\,d\theta$$

$$= \int 4\cos^2\theta\,d\theta$$

$$= \int 4\left(\frac{1+\cos 2\theta}{2}\right)d\theta$$

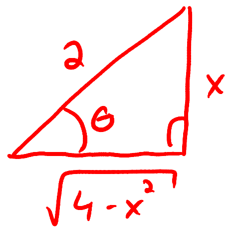
$$= \int 2(1+\cos 2\theta)\,d\theta$$

$$= 2\left(\theta + \frac{\sin 2\theta}{2}\right) + C$$

$$= 2\theta + \sin 2\theta + C$$

but $x = 2\sin\theta$

$\sin\theta = \dfrac{x}{2}$




$\theta = \sin^{-1}\left(\dfrac{x}{2}\right)$

$\sin 2\theta = 2\sin\theta\cos\theta = \cancel{2}\left(\dfrac{x}{\cancel{2}}\right)\left(\dfrac{\sqrt{4-x^2}}{2}\right)$

$$= 2\sin^{-1}\left(\frac{x}{2}\right) + \frac{x\sqrt{4-x^2}}{2} + C$$