Section 28.8 : cont'd

Tuesday, February 19, 2013
10:32 AM

integration by trig substitution:

guidelines:

$a^2 - x^2 \rightarrow x = a \sin\theta$

$a^2 + x^2 \rightarrow x = a \tan\theta$

$x^2 - a^2 \rightarrow x = a \sec\theta$

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evaluate:

$$\int \frac{\sqrt{x^2 - 25}}{x}\, dx$$

let $x = 5\sec\theta$
$dx = 5\sec\theta\tan\theta\, d\theta$

$$= \int \frac{\sqrt{25\sec^2\theta - 25}}{5\sec\theta} 5\sec\theta\tan\theta\, d\theta$$

$$= \int \sqrt{25(\sec^2\theta - 1)}\tan\theta\, d\theta$$

$$= \int \sqrt{25\tan^2\theta}\tan\theta\, d\theta$$

$$= \int 5\tan\theta\tan\theta\, d\theta$$

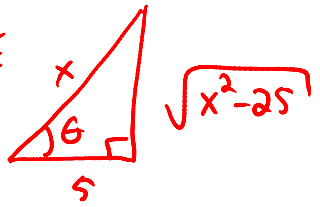
$$= \int 5\tan^2\theta\, d\theta$$

$$= \int 5(\sec^2\theta - 1)\, d\theta$$

$$= 5\tan\theta - 5\theta + C$$

$\sec\theta = \frac{x}{5}$

$\cos\theta = \frac{5}{x}$



$\tan\theta = \frac{\sqrt{x^2-25}}{5}$

$\theta = \cos^{-1}\frac{5}{x}$ or $= \sec^{-1}\frac{x}{5}$

$= \frac{5\sqrt{x^2-25}}{5} - 5\cos^{-1}\left(\frac{5}{x}\right) + C$

$= \sqrt{x^2-25} - 5\cos^{-1}\left(\frac{5}{x}\right) + C$

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evaluate $\int \frac{dx}{x\sqrt{9-x^2}}$

let $x = 3\sin\theta$

$dx = 3\cos\theta\, d\theta$

$$= \int \frac{3\cos\theta \, d\theta}{3\sin\theta \sqrt{9 - 9\sin^2\theta}}$$

$$= \int \frac{\cos\theta \, d\theta}{\sin\theta \sqrt{9\cos^2\theta}}$$

$$= \int \frac{\cos\theta \, d\theta}{3\sin\theta \cos\theta}$$

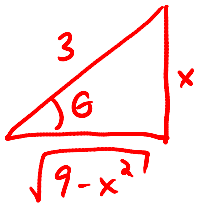
$$= \int \frac{d\theta}{3\sin\theta}$$

$$= \int \frac{1}{3}\csc\theta \, d\theta$$

$$= \frac{1}{3}\ln\left|\csc\theta - \cot\theta\right| + C$$

← top one on formula sheet

$\sin\theta = \frac{x}{3}$



$\csc\theta = \frac{3}{x}$

$\cot\theta = \frac{\sqrt{9-x^2}}{x}$

$$= \frac{1}{3}\ln\left|\frac{3}{x} - \frac{\sqrt{9-x^2}}{x}\right| + C$$

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this one's useful for partial fraction (next section)

$$\int \frac{1}{(x^2+4)^2} \, dx$$

let $x = 2\tan\theta$

$dx = 2\sec^2\theta \, d\theta$

$$= \int \frac{2\sec^2\theta\, d\theta}{(4\tan^2\theta + 4)^2}$$

$$= \int \frac{2\sec^2\theta\, d\theta}{(4\sec^2\theta)^2}$$

$$= \int \frac{2}{16}\frac{\sec^2\theta\, d\theta}{\sec^4\theta}$$

$$= \int \frac{1}{8}\frac{d\theta}{\sec^2\theta}$$

$$= \int \frac{1}{8}\cos^2\theta\, d\theta$$

← cos is raised to an even power → power-reducing