Section 28.8: cont'd

Wednesday, February 20, 2013
10:28 AM

$$\int \frac{1}{(x^2+4)^2}\,dx$$

let $x = 2\tan\theta$
$dx = 2\sec^2\theta\,d\theta$

$$= \int \frac{2\sec^2\theta\,d\theta}{(4\tan^2\theta + 4)^2}$$

$$= \int \frac{2\sec^2\theta\,d\theta}{(4\sec^2\theta)^2}$$

$$= \int \frac{2}{16}\frac{\sec^2\theta\,d\theta}{\sec^4\theta}$$

$$= \int \frac{1}{8}\frac{d\theta}{\sec^2\theta}$$

$$= \int \frac{1}{8}\cos^2\theta\,d\theta$$

← cos is raised to an even power → power-reducing

from last time ↑ (we stopped there)

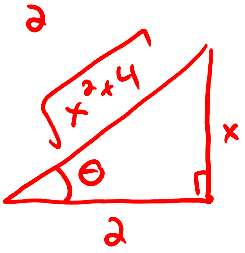
$$= \int \frac{1}{8}\frac{1+\cos 2\theta}{2}\,d\theta$$

$$= \int \frac{1}{16}(1+\cos 2\theta)\,d\theta$$

$$= \frac{1}{16}\left(\theta + \frac{\sin 2\theta}{2}\right) + C$$

let $x = 2\tan\theta$

$\tan\theta = \frac{x}{2}$



$\theta = \tan^{-1}\left(\frac{x}{2}\right)$

$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2 \frac{x}{\sqrt{x^2+4}} \frac{2}{\sqrt{x^2+4}}$

$= \frac{4x}{x^2+4}$

$= \frac{1}{16}\left(\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{2}\frac{4x}{x^2+4}\right) + C$

$= \frac{1}{16}\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{8}\frac{x}{x^2+4} + C$