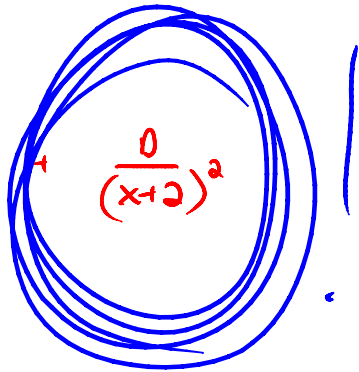


Section 28.10: Integration by Partial Fractions: Other Cases

Thursday, February 21, 2013
10:59 AM

repeated linear factors:

$$\frac{7x+10}{(x+1)(x+2)^2} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{0}{(x+2)^2}$$


horrific general case: (we won't give something this awful!)

$$\frac{5x^2+15x+1}{(x+1)^2(x+2)^3} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{D}{x+2} + \frac{E}{(x+2)^2} + \frac{F}{(x+2)^3}$$

not so nasty specific case:

$$\frac{7x+10}{(x+1)(x+2)^2} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{D}{(x+2)^2}$$

$$7x+10 = A(x+2)^2 + B(x+1)(x+2) + D(x+1)$$

let $x = -1$: $3 = A$

$x = -2$: $-4 = -D$ so $D = 4$

$x = 0$: $10 = 4A + 2B + D$

$10 = 12 + 2B + 4$

$B = -3$

↖ "dee"

so

$$\int \frac{7x+10}{(x+1)(x+2)^2} dx = \int \left(\frac{3}{x+1} - \frac{3}{x+2} + \frac{4}{(x+2)^2} \right) dx$$

$$= 3 \ln|x+1| - 3 \ln|x+2| - \frac{4}{x+2} + C$$

$$\begin{aligned} \text{note: } \int \frac{4}{(x+2)^2} dx &= \int 4(x+2)^{-2} dx \\ &= -4(x+2)^{-1} + c \end{aligned}$$

non-repeated quadratic factors

$$\frac{7x^2 - x - 17}{(x-1)(x^2+4)} = \frac{A}{x-1} + \frac{Bx+D}{x^2+4}$$