Section 28.10: cont'd

Monday, February 25, 2013
10:33 AM

non-repeated quadratic factors: (plus! repeated linear)

$$\frac{x^3 - x^2 + 24x + 16}{(x-2)^2(x^2+4x+5)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{Dx+E}{x^2+4x+5}$$

repeated linear

quadratic

note: would have to complete the square → $\tan^{-1}$ form

worst-case scenario (we would never ask one this ugly!)

$$\frac{5x-2}{(x+1)(x-3)^3(x^2+4x+5)^2} = \frac{A}{x+1} + \frac{B}{x-3} + \frac{D}{(x-3)^2} + \frac{E}{(x-3)^3} + \frac{Fx+G}{x^2+4x+5} + \frac{Hx+J}{(x^2+4x+5)^2}$$

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full example:

$$\int \frac{m^2+m-1}{(m^2+1)(m-2)}\,dm$$

partial fraction:

$$\frac{m^2+m-1}{(m^2+1)(m-2)} = \frac{Am+B}{m^2+1} + \frac{D}{m-2}$$

$$m^2+m-1 = (Am+B)(m-2) + D(m^2+1)$$

let $m = 2$: $\quad 5 = 5D$

$D = 1$

let $m = 0$: $\quad -1 = -2B + D$

$-2 = -2B$

$B = 1$

let $m = 1$: $\quad 1 = (A+B)(-1) + 2D$

$1 = -A - 1 + 2$

$A = 0$

$$\int \frac{m^2+m-1}{(m^2+1)(m-2)}\,dm = \int \left[\frac{1}{m^2+1} + \frac{1}{m-2}\right] dm$$

$$= \tan^{-1} m + \ln|m-2| + C$$

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another full example:

$$\int_3^4 \frac{5x^3-4x}{x^4-16}\,dx$$

partial fractions: $\quad x^4-16 = (x^2-4)(x^2+4)$

$= (x-2)(x+2)(x^2+4)$

$$\frac{5x^3-4x}{x^4-16} = \frac{A}{x-2} + \frac{B}{x+2} + \frac{Dx+E}{x^2+4}$$

$$5x^3-4x = A(x+2)(x^2+4) + B(x-2)(x^2+4) + (Dx+E)(x-2)(x+2)$$

let $x = -2$:

$-32 = -32B$

$B = 1$

let $x = 2$:

$32 = 32A$

$A = 1$

let $x = 0$:

$0 = 8A' - 8B' - 4E$

$E = 0$

let $x = 1$:

$1 = 15A' - 5B' - 3D$ (and $E=0$)

$-9 = -3D$

$D = 3$

$$\int_3^4 \frac{5x^3-4x}{x^4-16}\,dx = \int_3^4 \left[\frac{1}{x-2} + \frac{1}{x+2} + \frac{3x}{x^2+4}\right] dx$$

$$= \left[\ln|x-2| + \ln|x+2| + \frac{3}{2}\ln|x^2+4|\right]\Big|_3^4$$

$$= \left[\ln 2 + \ln 6 + \tfrac{3}{2}\ln 20\right] - \left[\ln 1 + \ln 5 + \tfrac{3}{2}\ln 13\right]$$

$$= \frac{1}{2}\ln\left(\frac{46080}{2197}\right)$$

$$\approx 1.522$$