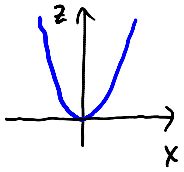
Section 29.2: Curves and Surfaces in Three Dimensions

Tuesday, February 26, 2013
10:34 AM

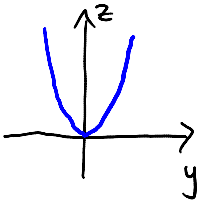
okay, so what does $f(x,y) = x^2 + y^2$ look like?

$z = x^2 + y^2$ ← 3 dimensional graph

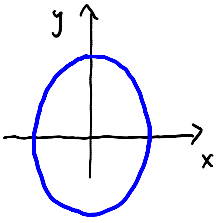
let $y=0$

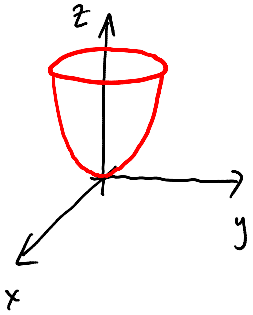


let $x=0$



let z = positive const

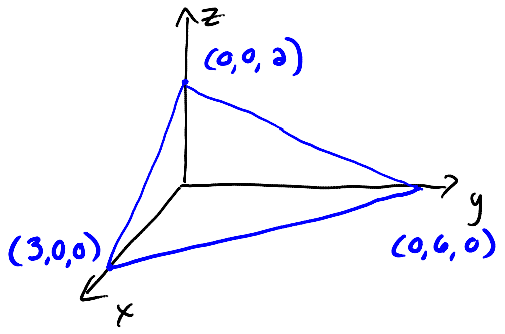




what about $ax + by + cz = d$ where $a, b, c,$ and d are constants?

plane

example: sketch: $2x + y + 3z = 6$



← the plane of the equation lies on top of this triangle

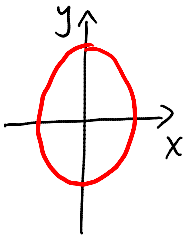
note: this triangle is the shape of the surface of the equation in the first octant

↑ like the first quadrant for two dimensions

→ octant is for 3D

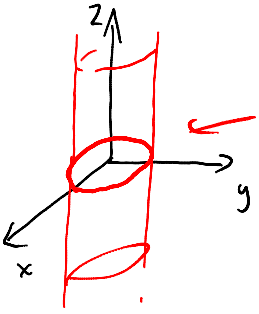
2D vs 3D:

in 2D, $x^2+y^2=4$ is



in 3D, $x^2+y^2=4$ is

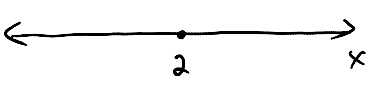
a cylinder



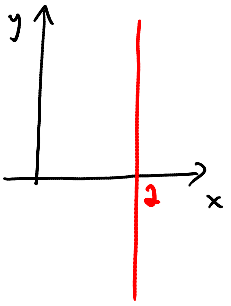
← circle in x-y plane

1D vs 2D vs 3D:

$x=2$ 1D



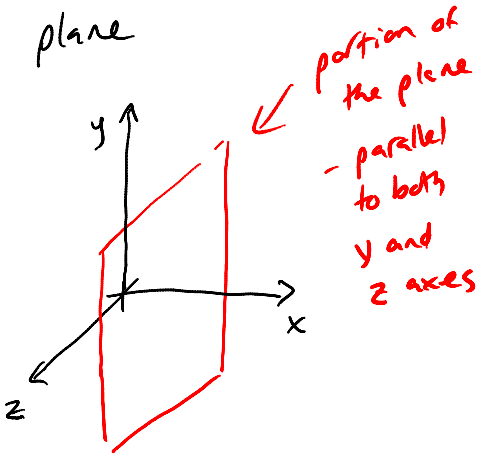
2D



3D: plane

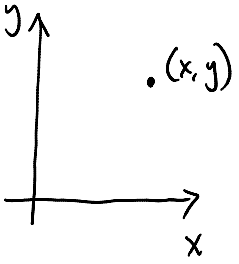
3D:

plane

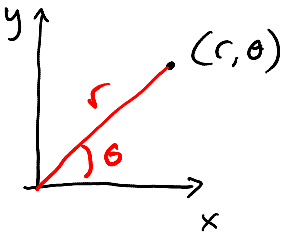


coordinate systems:

2D:



rectangular



polar

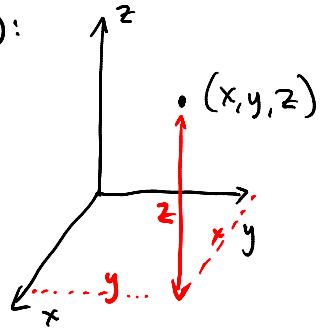
where $x = r\cos\theta$

$y = r\sin\theta$

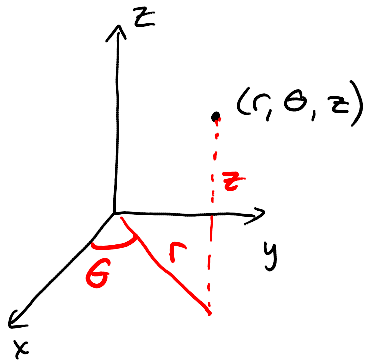
$r = \sqrt{x^2 + y^2}$

$\tan\theta = \frac{y}{x}$

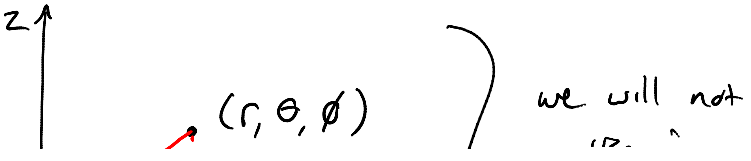
3D:



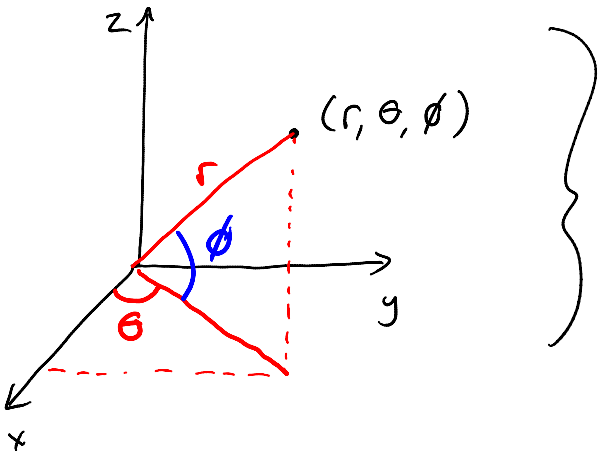
rectangular



cylindrical



we will not



we will not use in this course

Spherical

example: rewrite the following equation using cylindrical coordinates:

$$z = x^2 + y^2$$

recall: cylindrical is (r, θ, z) so leave z unchanged and substitute in for x and y

$$x = r\cos\theta$$
$$y = r\sin\theta$$

$$\begin{aligned} z &= x^2 + y^2 \\ &= r^2\cos^2\theta + r^2\sin^2\theta \\ &= r^2(\cos^2\theta + \sin^2\theta) \\ &= r^2 \end{aligned}$$

↗ paraboloid

note: there are no recommended problems from this section