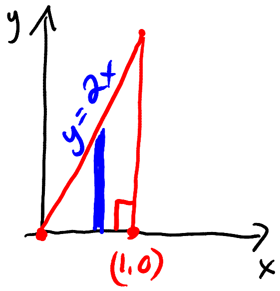
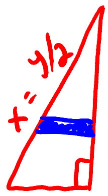
Section 29.4: cont'd

Friday, March 01, 2013
10:31 AM

example: evaluate the integral $I = \iint_T xy \, dA$ over the triangle T with vertices (0,0), (1,0), and (1,2).

step 1: draw the region in the xy plane





step 2: set up limits:

method #1:

T:

$0 \le x \le 1$

$0 \le y \le 2x$

method #2:

T:

$0 \le y \le 2$

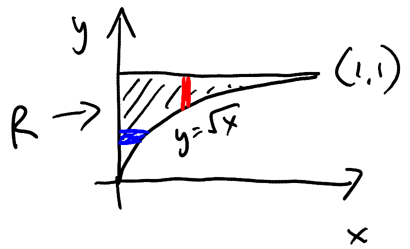
$y/2 \le x \le 1$

$$I = \iint_T xy\, dA$$
$$= \int_0^1 \int_0^{2x} xy\ dy\ dx$$
$$= \int_0^1 \left[\frac{xy^3}{2} \Big|_0^{2x} \right] dx$$
$$= \int_0^1 \left[2x^3 - 0 \right] dx$$
$$= \frac{2x^4}{4} \Big|_0^1$$
$$= 1/2$$

$$I = \iint_T xy\ dA$$
$$= \int_0^2 \int_{y/2}^1 xy\ dx\ dy$$
$$= \int_0^2 \left[\frac{x^2 y}{2} \Big|_{y/2}^1 \right] dy$$
$$= \int_0^2 \left[\frac{y}{2} - \frac{y^3}{8} \right] dy$$
$$= \left(\frac{y^2}{4} - \frac{y^4}{32} \right) \Big|_0^2$$
$$= 1 - \frac{1}{2}$$
$$= 1/2$$

example:

Find the volume of the region that lies below the graph of $z = e^{y^3}$ and above R:



method #1:

method #2:

$0 \le x \le 1$
$\sqrt{x} \le y \le 1$

$$V = \int_0^1 \underbrace{\int_{\sqrt{x}}^1 e^{y^3}\, dy}_{\text{nasty!}}\, dx$$

$0 \le y \le 1$
$0 \le x \le y^2$

$$V = \int_0^1 \int_0^{y^2} e^{y^3}\, dx\, dy$$

$$= \int_0^1 \left[x e^{y^3} \Big|_0^{y^2} \right] dy$$

$$= \int_0^1 y^2 e^{y^3}\, dy$$

$$= \frac{e^{y^3}}{3} \Big|_0^1$$

$$= \frac{e^1 - e^0}{3}$$

$$= \frac{e-1}{3}$$