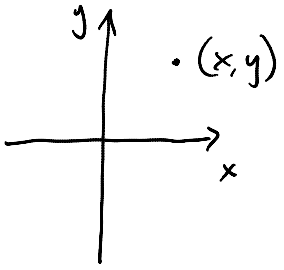
Section 29.4: Supplement on Cylindrical Coordinates

Friday, March 01, 2013
11:04 AM

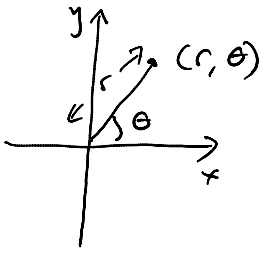
optional (but sometimes this makes the question much easier!)

rectangular coords



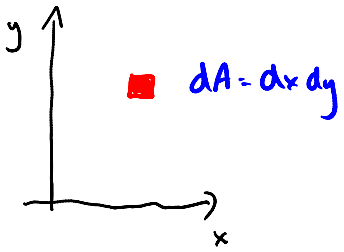
$x = r\cos\theta$
$y = r\sin\theta$

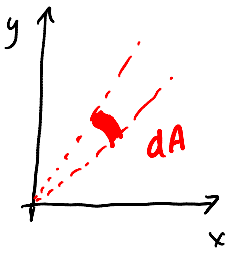
polar coords

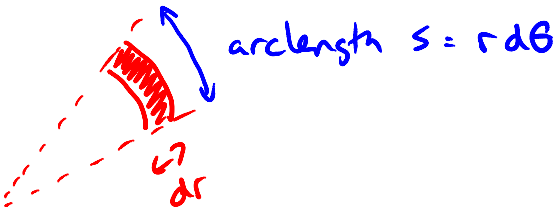


$r = \sqrt{x^2+y^2}$
$\tan\theta = y/x$

what about integrals?

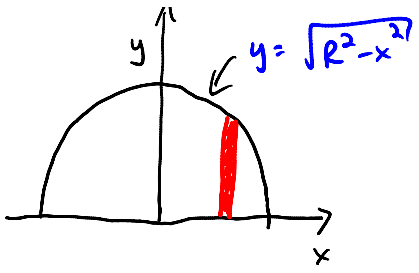






$dA = (r\,d\theta)(dr)$
$= r\,dr\,d\theta$

so, how does this work?



Find the area of the semicircle. Radius is R.

→ find the area of the first quadrant and multiply by 2

method #1:

$0 \le x \le R$
$0 \le y \le \sqrt{R^2 - x^2}$

$$A = \int_A dA$$

$$= \int_0^R \int_0^{\sqrt{R^2-x^2}} dy\,dx$$

$$= \int_0^R y \Big|_0^{\sqrt{R^2-x^2}} dx$$

$$= \int_0^R \sqrt{R^2 - x^2}\,dx$$

↑
trig sub
let $x = R\sin\theta$

$$= \int_{x=0}^{x=R} R\cos^2\theta\,d\theta$$

↓ power-reducing

$$= \frac{\pi R^2}{4}$$

so $A_{tot} = \frac{\pi R^2}{2}$

method #2:

$0 \le r \le R$
$0 \le \theta \le \pi/2$

$$A = \int_A dA$$

$$= \int_0^{\pi/2} \int_0^R r\,dr\,d\theta$$

$$= \int_0^{\pi/2} \frac{r^2}{2} \Big|_0^R d\theta$$

$$= \int_0^{\pi/2} \frac{R^2}{2} d\theta$$

$$= \frac{R^2}{2}\theta \Big|_0^{\pi/2}$$

$$= \frac{\pi R^2}{4}$$

A_{tot} = twice that!