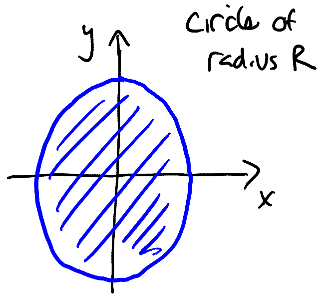
Section 29.4: supplement, cont'd

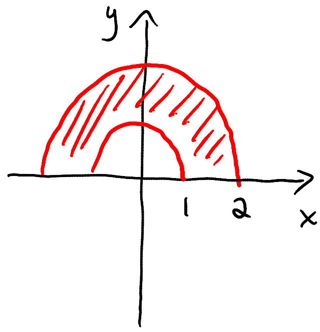
OPTIONAL!

Monday, March 04, 2013
10:34 AM

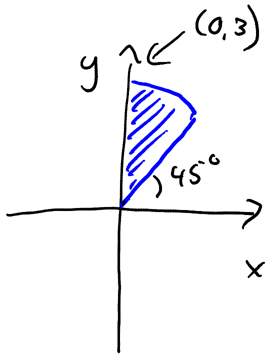
examples in setting up limits in polar coordinates:



$0 \le r \le R$
$0 \le \theta \le 2\pi$

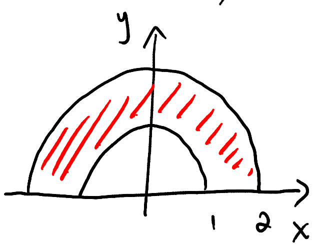


$1 \le r \le 2$
$0 \le \theta \le \pi$



$0 \le r \le 3$
$\pi/4 \le \theta \le \pi/2$

example: find the centre-of-mass of the following thin, uniform plate (two arcs are semicircles centred on the origin)



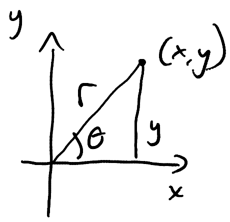
by symmetry, $\bar{x} = 0$

$$\bar{y} = \frac{1}{A} \int_A y \, dA$$

$$= \frac{1}{A} \int_0^{\pi} \int_1^2 y \; r \, dr \, d\theta$$

$$= \frac{1}{A} \int_0^{\pi} \int_1^2 r \sin\theta \; r \, dr \, d\theta$$

$$= \frac{1}{A} \int_0^{\pi} \int_1^2 r^2 \sin\theta \; dr \, d\theta$$



$$= \frac{1}{A}\int_0^{\pi}\left[\frac{r^3}{3}\sin\theta \Big|_1^2\right] d\theta$$

$$= \frac{1}{A}\int_0^{\pi} \sin\theta\left[\frac{8}{3}-\frac{1}{3}\right] d\theta$$

$$= \frac{1}{A}\int_0^{\pi} \frac{7}{3}\sin\theta \, d\theta$$

$$= \frac{1}{A}\frac{7}{3}\left(-\cos\theta\right)\Big|_0^{\pi}$$

$$= \frac{7}{3A}\left(-\cos\pi + \cos 0\right)$$

$$= \frac{7}{3A}\left(1 + 1\right)$$

$$= \frac{14}{3A}$$

$$\rightarrow A = \tfrac{1}{2}\pi r_2^2 - \tfrac{1}{2}\pi r_1^2$$
$$= \tfrac{1}{2}\pi(4-1)$$
$$= \frac{3}{2}\pi$$

$$= \frac{28}{9\pi}$$

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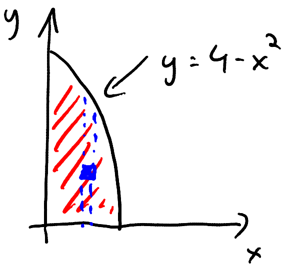
so, the book shows you how to evaluate double integrals, but what are they <u>actually</u> used for?

centre-of-mass: find the centre-of-mass of the thin uniform plate shown below



$dA = dx\,dy$ or $dy\,dx$

$0 \le x \le 2$

$0 \le y \le 4 - x^2$

$$A = \int_A dA = \int_0^2 \int_0^{4-x^2} dy\, dx$$

$$\bar{x} = \frac{1}{A} \int_A x_e \, dA = \frac{1}{A} \int_0^2 \int_0^{4-x^2} x \, dy\, dx$$

$$\bar{y} = \frac{1}{A} \int_A y_e \, dA = \frac{1}{A} \int_0^2 \int_0^{4-x^2} y \, dy\, dx$$