Section 30.1: Review of Sequences/Series cont'd

Tuesday, March 05, 2013
10:31 AM

Assignment #4 due on

Wednesday, March 13

Notation:

n^{th} term is a_n with n starting at 1

sequence: $a_1, a_2, a_3, \ldots, a_{n-1}, a_n$

series: $S_n = \sum_{i=1}^{n} a_i$

geometric series:

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

but what about

$$\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \ldots \quad ?$$

ratio: constant that you are multiplying by: $r = 1/4$

how do we calculate it?

naively plug into S_n:

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_\infty = \frac{1/4\left(1-(1/4)^\infty\right)}{1-1/4}$$

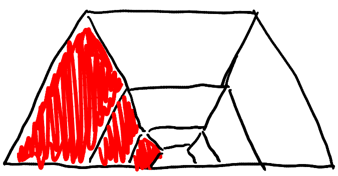
but what is $(1/4)^\infty$

$$\lim_{n\to\infty}\left(\frac{1}{4}\right)^n \to 0$$

provided that $|r| < 1$

$$S_\infty = \frac{1/4}{3/4} = \frac{1}{4}\cdot\frac{4}{3} = \frac{1}{3}$$

general case: $S_\infty = \dfrac{a_1}{1-r}$ provided that $|r|<1$



examples:

evaluate: $100 + 20 + 4 + \ldots$

$a_1 = 100$

$r = 1/5$

$|r|<1$? ✓

$$S_\infty = \frac{a_1}{1-r} = \frac{100}{1-1/5} = \frac{100}{4/5} = 100\cdot\frac{5}{4} = 125$$

evaluate: $48 - 64 + \frac{256}{3} - \ldots$

$a_1 = 48$

$r = \frac{\text{any term}}{\text{previous term}} = \frac{-64}{48} = -\frac{4}{3}$

but $|r| \cancel{<} 1$? NO!

$S_\infty =$ undefined

evaluate: $1 - 1 + 1 - 1 + 1 - 1 + 1 \ldots$

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what about:

$1 + x + x^2 + x^3 + \ldots$ for $-1 < x < 1$

$a_1 = 1$

$r = x$

$S_\infty = \frac{a_1}{1-r} = \frac{1}{1-x}$

but

$$
\begin{array}{r|l}
 & 1 + x + x^2 + x^3 \ldots \\
\hline
1 - x & 1 \\
 & \underline{1 - x} \\
 & \quad x \\
 & \quad \underline{x - x^2} \\
 & \qquad x^2 \\
 & \qquad \underline{x^2 - x^3} \\
 & \qquad\quad x^3
\end{array}
$$

summary:

- infinite series can under certain conditions have a finite sum

Calculus idea:

- often useful to work backwards and replace a single term (function) by its series expansion

then approximate by taking the first few terms