# Section 30.2/30.3/30.4: contd

Wednesday, March 06, 2013
10:39 AM

definition of Maclaurin series:

$$f(x) = f(0) + \frac{f'(0)\,x}{1!} + \frac{f''(0)\,x^2}{2!} + \frac{f'''(0)\,x^3}{3!} + \dots$$

→ this expansion is about $x=0$, so will work even taking just the first few terms will work well when $x$ is small

---

example: use the definition of Maclaurin series to find the first four terms of the expansion for $f(x) = e^x$

| | | | |
|---|---|---|---|
| $f(x) = e^x$ | | $f(0) = 1$ | |
| $f'(x) = e^x$ | | $f'(0) = 1$ | |
| $f''(x) = e^x$ | | $f''(0) = 1$ | |
| $f'''(x) = e^x$ | | $f'''(0) = 1$ | |

$$f(x) = f(0) + \frac{f'(0)\,x}{1!} + \frac{f''(0)\,x^2}{2!} + \frac{f'''(0)\,x^3}{3!} + \dots$$

$$\boxed{= 1 + x + \frac{x^2}{2} + \frac{x^3}{6} +} \dots$$

note: $5! = 5\cdot4\cdot3\cdot2\cdot1$
$3! = 3\cdot2\cdot1$
$1! = 1$
$0! = 1$

so, how accurate is this?

example: use the first four terms of the Maclaurin series for $f(x) = e^x$ to estimate.

to estimate

a) $e^{0.2}$

b) $e^{1}$

and compare with the result from your calculator (round to 4 decimals)

a) $e^x = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \dots$

$$e^{0.2} \approx 1 + 0.2 + \frac{(0.2)^2}{2} + \frac{(0.2)^3}{6}$$

$\approx 1.22133$

$\approx 1.2213$

note: calculator says $e^{0.2} \approx 1.2214$

b) $$e^{1} \approx 1 + 1 + \frac{1}{2} + \frac{1}{6}$$

$\approx 2.\bar{6}$

note: calculator says $e^{1} = 2.7182\ldots$

---

so, when would we actually use this?

example: Use the first three terms of the Maclaurin series for $e^x$ to estimate the following integral to three decimal places.

$$\int_{0.1} \frac{e^x}{x}\,dx = \int_{0.1} \frac{1}{x}\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots\right) dx$$

$$\approx \int_{0.1}^{0.2} \frac{1}{x}\left(1 + x + \frac{x^2}{2}\right) dx$$

$$\approx \int_{0.1}^{0.2} \left(\frac{1}{x} + 1 + \frac{x}{2}\right) dx$$

$$\approx \left(\ln|x| + x + \frac{x^2}{4}\right)\Big|_{0.1}^{0.2}$$

$$\approx \left(\ln 0.2 + 0.2 + \frac{0.2^2}{4}\right) - \left(\ln 0.1 + 0.1 + \frac{0.1^2}{4}\right)$$

$$\approx 0.800647$$

$$\approx 0.801$$

Ti-89 says 0.801052

Summary: Maclaurin series

What is a Maclaurin series?

- a method to generate a series expansion (polynomial) for any function about $x=0$

note: the function and its derivatives must exist at $x=0$

Why do we care?

- can use for estimates (in the age of calculators, not as useful as in the past)

- replace the original function → say, in a nasty integral!

note: Maclaurin expands about $x=0$

note: Maclaurin expands about $x=0$

→ if you are far from zero, use Taylor series (section 30.5) instead

---

one last example: Use first four terms of the Maclaurin series for $e^x$ to generate expansions for

a) $e^{-x^2}$

b) $\frac{e^{-x^2}}{x}$

$$e^x \approx 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$$

$$e^{-x^2} \approx 1 + (-x^2) + \frac{(-x^2)^2}{2!} + \frac{(-x^2)^3}{3!}$$

$$\approx 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!}$$

$$\frac{e^{-x^2}}{x} \approx \frac{1}{x} - x + \frac{x^3}{2!} - \frac{x^5}{3!}$$