Sections 30.2/30.3/30.4: Cont'd

Thursday, March 07, 2013
10:33 AM

okay, enough about e^x. How about:

Maclaurin series for $\sin x$:

example: Using the definition of Maclaurin series, find the first four non-zero terms for $f(x) = \sin x$.

$$\left.\begin{array}{llll} f(x) = \sin x & & & f(0) = 0 \\ f'(x) = \cos x & & & f'(0) = 1 \\ f''(x) = -\sin x & & & f''(0) = 0 \\ f'''(x) = -\cos x & & & f'''(0) = -1 \end{array}\right\} \text{sets up a pattern}$$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \ldots$$

$$= 0 + x + \frac{0x^2}{2!} + \frac{(-1)x^3}{3!} + \ldots$$

$$= \underbrace{x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}}_{\text{terms have alternating signs}} + \ldots$$

$\Rightarrow$ this series is called an alternating series

Maclaurin series of $\cos x$:

note: could use the same method as we used for $\sin x$, or can take a shortcut example: Use differentiation of the Maclaurin series for $\sin x$ to find the Maclaurin series for $\cos x$ (find the first four non-zero terms).

$$\frac{d}{dx}(\sin x) = \frac{d}{dx}\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \cdots\right)$$

$$\cos x = 1 - \frac{3x^2}{3!} + \frac{5x^4}{5!} - \frac{7x^6}{7!} + \cdots$$

$$= \underbrace{1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots}$$

$\cos x$ is an even function and all of the terms in the expansion are even

$\sin x \Leftrightarrow$ odd terms

↑

odd symmetry

---

example: find the first six non-zero terms of the Maclaurin series for
$f(x) = (1+x)\sin x$

laziest possible method:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$(1+x)\sin x = (1+x)\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots\right)$$

$$= \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots\right)$$

$$+ \left(x^2 - \frac{x^4}{3!} + \frac{x^6}{5!} - \frac{x^8}{7!} + \cdots\right)$$

$$= x + x^2 - \frac{x^3}{3!} - \frac{x^4}{3!} + \frac{x^5}{5!} + \frac{x^6}{5!} + \cdots$$

Find the first four non-zero terms of the Maclaurin series for $f(x) = e^{2x} \cos x$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$e^{2x} = 1 + 2x + \frac{4x^2}{2!} + \frac{8x^3}{3!} + \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$e^{2x}\cos x = \left(1 + \underline{2x} + \underline{\underline{2x^2}} + \frac{4x^3}{3} + \cdots\right)\left(1 - \frac{x^2}{2} + \frac{x^4}{24} + \cdots\right)$$

$$= 1 - \frac{x^2}{2} + \frac{x^4}{24} + \cdots$$

$$+ 2x - \frac{2x^3}{2} + \frac{2x^5}{24} + \cdots$$

$$+ 2x^2 - \frac{2x^4}{2} + \cdots$$

$$+ \frac{4x^3}{3}$$

$$= 1 + 2x + \underline{3}\,x^2 + \underline{x^3} + \cdots$$

$$= 1 + 2x + \frac{3}{2}x^2 + \frac{x^3}{3} + \cdots$$