Sections 30.2/30.3/30.4: cont'd

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10:35 AM

so, yesterday we found that

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$$

What if we want to find:

$$\ln(1+x)$$

$\Leftarrow$ could certainly use definition of Maclaurin here (derivatives exist)

$$\frac{d}{dx}\left(\ln(1+x)\right) = \frac{1}{1+x}$$

$\uparrow$ can integrate this to get $\ln(1+x)$

idea: take the series for $\frac{1}{1-x}$, change it to $\frac{1}{1+x}$, and integrate it

$$\frac{1}{1+x} = \frac{1}{1-(-x)}$$

$$= 1 + (-x) + (-x)^2 + (-x)^3 + \dots$$

$$= 1 - x + x^2 - x^3 + \dots$$

so, integrate it:

$$\int \frac{1}{1+x}\,dx = \int \left(1 - x + x^2 - x^3 + \dots\right) dx$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \dots \quad + C$$

how to find C? set $x=0$ and solve for C

$$\ln 1 = C$$

$$C = 0$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \dots$$

note: how can you get the Maclaurin series for $\tan^{-1} x$?

$$\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$$

so modify the $\frac{1}{1+x}$ expansion and integrate, noting that $\tan^{-1} 0 = 0$ to find constant