Section 30.5: Taylor Series

Tuesday, March 12, 2013
10:52 AM

Maclaurin series: only accurate for values of x close to zero

- what if x isn't close to zero?

  → expand about some other value instead!

- power series expansion, but instead of x, all of our terms are in $(x-a)$

Taylor series:

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots$$

note: this one's on the final exam formula sheet

to get Maclaurin, set $a = 0$

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example: using the definition, find the first three non-zero terms of the Taylor series of

$$f(x) = e^{x-1} \text{ about } a = 2$$

$f(x) = e^{x-1}$ $\quad$ $f(2) = e$

$f'(x) = e^{x-1}$ $\quad$ $f'(2) = e$

$f''(x) = e^{x-1}$ $\quad$ $f''(2) = e$

$f'''(x) = e^{x-1}$ $\quad$ $f'''(2) = e$

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots$$

$$= e + e(x-2) + \frac{e}{2}(x-2)^2 + \cdots$$

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example: expand $\sqrt[4]{x}$ in powers of $x-16$.
Give the first three terms

$\underbrace{\quad}$ Taylor series

$f(x) = x^{1/4}$ $\qquad$ $f(16) = 2$

$f'(x) = \frac{1}{4} x^{-3/4}$ $\qquad$ $f'(16) = \frac{1}{32}$

$f''(x) = \frac{-3}{16} x^{-7/4}$ $\qquad$ $f''(16) = \frac{-3}{16} \cdot 2^{-7} = \frac{-3}{2048}$

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots$$

$$f(x) \approx 2 + \frac{1}{32}(x-16) - \frac{3}{4096}(x-16)^2$$

use the results of the previous example to estimate $\sqrt[4]{17}$ to 4 decimal places

$$\sqrt[4]{x} \approx 2 + \frac{1}{32}(x-16) - \frac{3}{4096}(x-16)^2$$

$$\sqrt[4]{17} \approx 2 + \frac{1}{32} - \frac{3}{4096}$$

$$\approx 2.03052$$

$$\approx 2.0305$$

calculator says 2.03054

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example: by using the first three non-zero terms of a Taylor series, evaluate $\cos 28°$.
Round your answer to 4 decimal places.

$$\cos 28^\circ = \cos(30^\circ - 2^\circ)$$

$$= \cos\left(\frac{\pi}{6} - \frac{\pi}{90}\right)$$ ⇐ expanding about $\pi/6$ so $x-a = -\pi/90$

$f(x) = \cos x$ $\quad$ $f(\pi/6) = \sqrt{3}/2$

$f'(x) = -\sin x$ $\quad$ $f'(\pi/6) = -1/2$

$f''(x) = -\cos x$ $\quad$ $f''(\pi/6) = -\sqrt{3}/2$

$$f(x) \approx f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2$$

$$\cos x \approx \frac{\sqrt{3}}{2} - \frac{1}{2}\left(x - \pi/6\right) - \frac{\sqrt{3}}{4}\left(x - \pi/6\right)^2$$

$$\cos 28^\circ \approx \frac{\sqrt{3}}{2} - \frac{1}{2}\left(-\pi/90\right) - \frac{\sqrt{3}}{4}\left(-\pi/90\right)^2$$

≈ 0.882951

≈ 0.8830

Calculator value:
0.882948