Review:

Wednesday, March 13, 2013
10:32 AM

Find the displacement $x(t)$ of an object if its acceleration is given by

$$a = 12t$$

where t is in seconds and a is in m/s^2. The initial velocity is $5\ m/s$.

$$v = \int a\,dt$$

$$= \int 12t\,dt$$

$$= 6t^2 + C$$

at $t = 0$, $v = 5\ m/s$

$$5\ m/s = 6 \cdot 0 + C \quad \text{so} \quad C = 5$$

$$x = \int (6t^2 + 5)\,dt$$

$$= 2t^3 + 5t + C_1$$

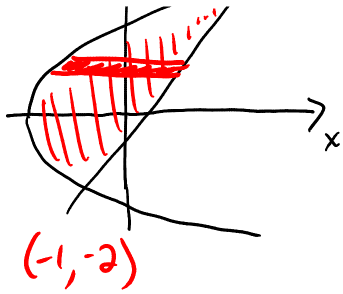
at $t = 0$, $x = 0$

$$\text{so} \quad C_1 = 0$$

$$x = 2t^3 + 5t$$

Find the area between the line $y = x - 1$ and the parabola $y^2 = 2x + 6$.

two curve intersect at:



$$y_1 = y_2$$

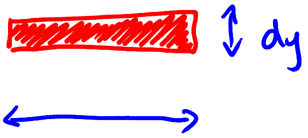
$$x - 1 = \sqrt{2x+6}$$

$$x^2 - 2x + 1 = 2x + 6$$

$$x^2 - 4x - 5 = 0$$

$$(x+1)(x-5) = 0$$

$$x = -1, 5$$



$$\ell = x_{right} - x_{left}$$

$$= y + 1 - \frac{y^2 - 6}{2}$$

$$y = x - 1 \qquad x = y + 1 \quad \text{(right)}$$

$$y^2 = 2x - 6$$

$$x = \frac{y^2 - 6}{2} \quad \text{(left)}$$

$$dA = \ell\, dy$$

$$= \left(y + 1 - \frac{y^2 - 6}{2}\right) dy$$

$$A = \int_A dA$$

$$= \int_{-2}^{4} \left(y + 1 - \left(\frac{y^2 - 6}{2}\right)\right) dy$$

$$= \int_{-2}^{4} \left(-\frac{y^2}{2} + y + 4\right) dy$$

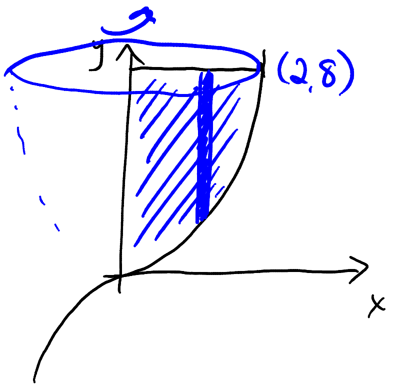
$$= \left(-\frac{y^3}{6} + \frac{y^2}{2} + 4y\right)\Bigg|_{-2}^{4}$$

$$= -\frac{64}{6} + 8 + 16 - \left(\frac{8}{6} + 2 - 8\right)$$

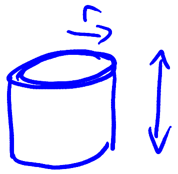
$$= 18$$

Find the moment of inertia about the y-axis for the following

Find the moment of inertia for the following solid of revolution: consider the region bounded by $y = x^3$, $x = 0$, and $y = 8$, rotated about the y-axis. You may leave your answer in terms of the density ρ.



$dV = 2\pi r h \, dt$



$h = y_{top} - y_{bottom} = 8 - x^3$

$dt = dx$ $\qquad r = x$

$$dV = 2\pi \, x \, (8 - x^3) \, dx$$

$$I = \rho \int_V r^2 \, dV$$

$$= \rho \int_0^2 x^2 \, 2\pi x (8 - x^3) \, dx$$

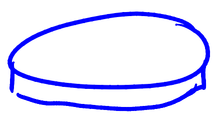
$$= 2\pi\rho \int_0^2 (8x^3 - x^6) \, dx$$

$$= 2\pi\rho \left(2x^4 - \frac{x^7}{7} \right) \Big|_0^2$$

$$= 2\pi\rho \left(32 - \frac{128}{7} \right)$$

$$= \frac{192}{7} \pi\rho \quad \text{or} \quad 27.4 \, \pi\rho$$

note: if using disks:



$dV = \pi r^2 dt$

$$dI = \tfrac{1}{2} m r^2, \text{ not } m r^2$$

$$I = \rho \int_V r_\perp^2 \, dV$$

$$= \rho \int \frac{r^2}{2} \, dV_{disk}$$