# Math 187 Review: Completing the Square

To complete the square, you need to perform three steps:

## Step 1:

To make $x^2 + 10x +$ ____ a perfect square, take the coefficient on the $x$ term and divide by 2, then square it. So 10/2 = 5, and the square is 25. Then $x^2 + 10x + 25$ is a perfect square: $x^2 + 10x + 25 = (x+5)^2$.

Exercises: fill in the blank so that the expression is a perfect square.

1. $x^2 + 6x +$ _____
2. $y^2 - 8y +$ _____
3. $x^2 + 12x +$ _____
4. $p^2 - 2p +$ _____
5. $w^2 - w +$ _____
6. $z^2 + 3z +$ _____

## Step 2:

Once you have made your perfect square, add an extra constant so that your new expression is equivalent to the original one. For example, $x^2 + 10x$ becomes $x^2 + 10x + 25$ as before, but now we have to subtract 25 so that $x^2 + 10x + 25 - 25$. Note that if we simplified, we'd get the $x^2 + 10x$ that we started with.

Try:

7. $b^2 + 14b$
8. $m^2 + 20m$
9. $x^2 - 4x$
10. $v^2 + 8v$
11. $r^2 - 5r$
12. $h^2 + h$

## Step 3:

Do steps 1 and 2 as above, but now factor the first three terms of your expression. If you completed step one correctly, the first three terms are automatically a perfect square trinomial.

Hint: do you remember what you got when you divided the middle coefficient by 2? Add that to your variable and you'll get the term that will be squared. For example, $x^2 + 10x + 25 - 25$ becomes $(x+5)^2 - 25$ because we divided 10/2 to get 5. And $x^2 - 10x + 25 - 25$ becomes $(x-5)^2 - 25$ because we divided –10/2 to get –5.

13. $a^2 - 6a$
14. $b^2 + 16b$
15. $k^2 + 2k$
16. $q^2 - 18q$
17. $x^2 + 3x$
18. $w^2 - 5w$

## What if the leading coefficient isn't 1?

Extra step: factor out the leading coefficient (the coeff on the squared term) before making the expression a perfect square. Make sure that you multiply the constant you added by the coefficient before adding the second balancing constant. For example: to complete the square for $3x^2+30x$, first factor out the leading coeff: $3\left(x^2+10x\right)$. Then complete the square inside the brackets: $3\left(x^2+10x+25\right)$. But we can't just add 25 to balance: the 25 is multiplied by the leading coefficient, so to balance it we must subtract the product: $3\left(x^2+10x+25\right)-3(25)$ which finally gives us $3(x+5)^2-75$. Note that if you expand this expression and simplify, you get back the original $3x^2+30x$.

Exercises: complete the square.

19. $2y^2-12y$
20. $-a^2+4a$
21. $7z^2+42z$
22. $-4t^2-12t$
23. $3m^2-m$
24. $-5x^2+120x$

## Answers

1. $x^2+6x+9$
2. $y^2-8y+16$
3. $x^2+12x+36$
4. $p^2-2p+1$
5. $w^2-w+\frac{1}{4}$
6. $z^2+3z+\frac{9}{4}$
7. $b^2+14b+49-49$
8. $m^2+20m+100-100$
9. $x^2-4x+4-4$
10. $v^2+8v+16-16$
11. $r^2-5r+\frac{25}{4}-\frac{25}{4}$
12. $h^2+h+\frac{1}{4}-\frac{1}{4}$
13. $(a-3)^2-9$
14. $(b+8)^2-64$
15. $(k+1)^2-1$
16. $(q-9)^2-81$
17. $\left(x+\frac{3}{2}\right)^2-\frac{9}{4}$
18. $\left(w-\frac{5}{2}\right)^2-\frac{25}{4}$
19. $2(y-3)^2-18$
20. $-(a-2)^2+4$
21. $7(z+3)^2-63$
22. $-4\left(t+\frac{3}{2}\right)^2+9$
23. $3\left(m-\frac{1}{2}\right)^2-\frac{3}{4}$
24. $-5(x-12)^2+720$