Math 187 Review: Trig Substitution

Here is a typical question that we need to answer in order to properly integrate by trig substitution: if we've decided that $x = 4\tan\theta$, then what are θ, $\sin\theta$ and $\sec\theta$ in terms of *x*?

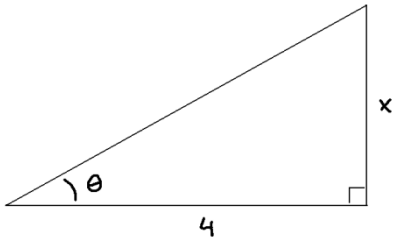
To solve this problem, we need a few steps.

Step 1:

Isolate the trig identity in the equation, and sketch the related triangle.

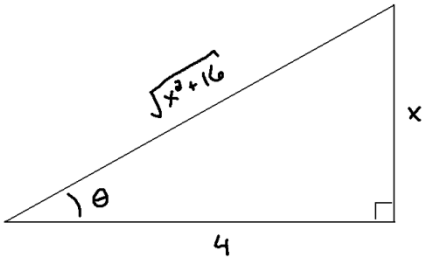
For example, if $x = 4\tan\theta$, then $\tan\theta = \frac{x}{4}$. Note that if we need to find θ, it's just $\theta = \tan^{-1}\left(\frac{x}{4}\right)$.

Since $\tan\theta = \frac{x}{4} = \frac{opp}{adj}$, then *x* is the opposite side and 4 the adjacent side. The related triangle is shown below.



Step 2:

Calculate the remaining side of the triangle. Since it's a right triangle, we can use Pythagorus to determine that if $a^2 + b^2 = c^2$, then $x^2 + 4^2 = c^2$ and $c = \sqrt{x^2 + 16}$.



Step 3:

Write down the ratios of interest by examining your finished triangle.

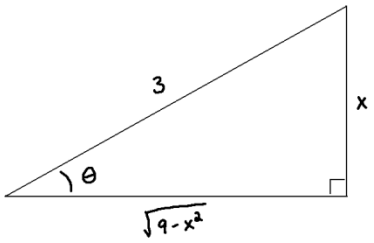
For example, $\sin\theta = \frac{opp}{hyp} = \frac{x}{\sqrt{x^2+16}}$. To find the secant, first find the cosine, $\cos\theta = \frac{adj}{hyp} = \frac{4}{\sqrt{x^2+16}}$, and then take the reciprocal: $\sec\theta = \frac{1}{\cos\theta} = \frac{\sqrt{x^2+16}}{4}$.

Exercises

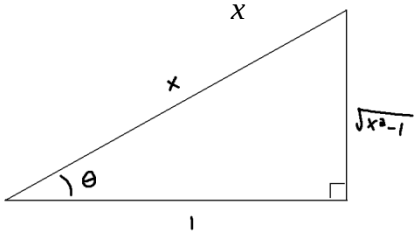
1. If $x = 3\sin\theta$, find θ, $\cos\theta$ and $\tan\theta$ in terms of *x*.
2. If $x = \sec\theta$, find $\sin\theta$, $\tan\theta$, and $\cot\theta$ in terms of *x*.
3. If $x = a\tan\theta$ where *a* is a positive constant, find $\cos\theta$ and $\sec\theta$ in terms of *x* and *a*.
4. If $x - 1 = \sin\theta$, find $\csc\theta$ and $\tan\theta$ in terms of *x*.
5. If $x = \frac{1}{2}\sec\theta$, find $\sin\theta$, $\cos\theta$ and $\tan\theta$ in terms of *x*.

Answers

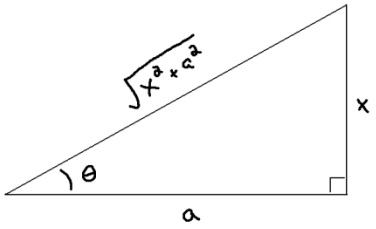
1. Triangle is shown below. $\theta = \sin^{-1}\left(\frac{x}{3}\right)$, $\cos\theta = \frac{\sqrt{9-x^2}}{3}$ and $\tan\theta = \frac{x}{\sqrt{9-x^2}}$



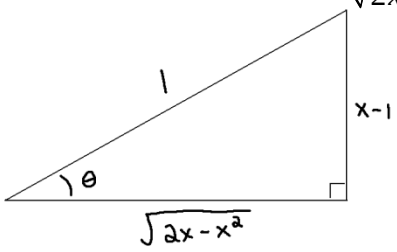
2. Triangle is shown below. $\sin\theta = \frac{\sqrt{x^2-1}}{x}$, $\tan\theta = \sqrt{x^2-1}$, and $\cot\theta = \frac{1}{\sqrt{x^2-1}}$



3. Triangle is shown below. $\cos\theta = \dfrac{a}{\sqrt{x^2+a^2}}$ and $\sec\theta = \dfrac{\sqrt{x^2+a^2}}{a}$



4. Triangle is shown below. $\csc\theta = \dfrac{1}{x-1}$ and $\tan\theta = \dfrac{x-1}{\sqrt{2x-x^2}}$



5. Triangle is shown below. $\sin\theta = \dfrac{\sqrt{4x^2-1}}{2x}$, $\cos\theta = \dfrac{1}{2x}$ and $\tan\theta = \sqrt{4x^2-1}$