Math 187 – Quiz #1

January 25, 2013 **Name:** Solution Set
Instructor: Patricia Wrean & Gilles Cazelais

Total: 25 points

1. Evaluate the following integrals. (6 points)

a) $\int_1^2 \left(12x^5 - 4x\right) dx$ $\underline{120}$

$$= \left(\frac{12x^6}{6} - \frac{4x^2}{2}\right)\Big|_1^2$$

$$= \left(2x^6 - 2x^2\right)\Big|_1^2$$

$$= \left(2^7 - 2^3\right) - (0)$$

$$= 120$$

b) $\int \left(3m\sqrt{m} + 4\right) dm$ $\underline{\frac{6}{5}m^{5/2} + 4m + C}$

$$= \int \left(3m^{3/2} + 4\right) dm$$

$$= 3\,\frac{2}{5}m^{5/2} + 4m + C$$

$$= \frac{6}{5}m^{5/2} + 4m + C$$

c) $\int x^2 \sqrt[3]{8 - x^3}\, dx$ $\underline{-\frac{1}{4}\left(8 - x^3\right)^{4/3} + C}$

let $u = 8 - x^3$

$du = -3x^2 dx$

$\frac{du}{-3} = x^2 dx$

$$= \int u^{1/3} \frac{du}{-3}$$

$$= -\frac{1}{3}\cdot\frac{3}{4}u^{4/3} + C$$

$$= -\frac{1}{4}\left(8 - x^3\right)^{4/3} + C$$

2. Approximate the integral $\int_2^3 \frac{\sqrt{x+1}}{x}\,dx$ with $n=4$ using Simpson's rule. Round your answer to three decimal places. (4 points)

x	$y = \frac{\sqrt{x+1}}{dx}$
2	0.8660
2.25	0.8012
2.5	0.7483
2.75	0.7042
3	0.6667

$h = \frac{b-a}{n} = \frac{3-2}{4} = \frac{1}{4}$ or 0.25

Simpson's:

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + y_4\right)$$

$$\approx \frac{0.25}{3}\Big(0.8660 + 4(0.8012) + 2(0.7483) + 4(0.7042) + 0.6667\Big)$$

$$\approx 0.7542$$

$$\approx 0.759$$

note: actual integral gives 0.754244 when evaluated, so pretty good!

3. Find the equation of a curve $y = f(x)$ whose slope is $\frac{1}{(2x+3)^5}$. The curve passes through the point $(-1,4)$. (5 points)

$$\frac{dy}{dx} = \frac{1}{(2x+3)^5}$$

$$y = \int \frac{dy}{dx}\, dx$$

$$= \int \frac{1}{(2x+3)^5}\, dx$$

$$= \int u^{-5} \frac{du}{2}$$

let $u = 2x+3$

$du = 2dx$

$\frac{du}{2} = dx$

$$= \frac{1}{2} \frac{u^{-4}}{-4} + C$$

$$= -\frac{1}{8}(2x+3)^{-4} + C$$

at $x = -1$, $y = 4$:

$$4 = -\tfrac{1}{8}(1)^{-4} + C$$

$$C = 4 + \tfrac{1}{8} = \tfrac{33}{8}$$

$(= 4.125)$

so

$$\boxed{y = -\tfrac{1}{8}(2x+3)^{-4} + \tfrac{33}{8}}$$

4. Find the area bounded by $y = x^2 + 2x$ and $y = x + 2$. (5 points)

find coords of intersections:

$y_1 = y_2$

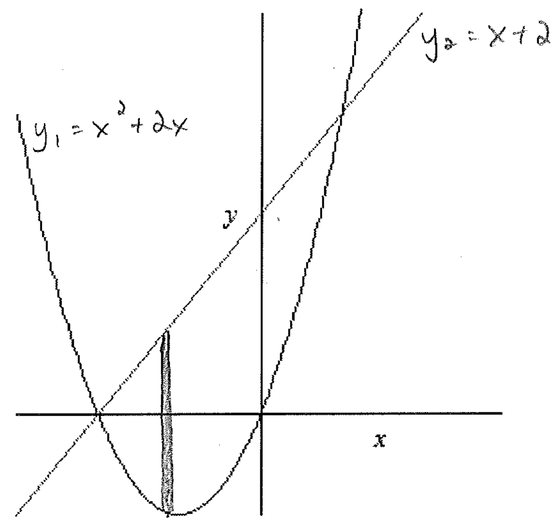
$x^2 + 2x = x + 2$

$x^2 + x - 2 = 0$

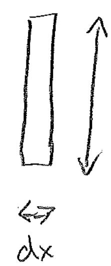
$(x+2)(x-1) = 0$

$x = -2, 1$

1



then



$y_2 - y_1 = (x+2) - (x^2 + 2x)$

$= 2 - x - x^2$

$$A = \int_A dA$$

$$= \int_{-2}^{1} (2 - x - x^2)\,dx$$

$dA = (y_2 - y_1)\,dx$

$= (2 - x - x^2)\,dx$

2

$$= \left(2x - \frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_{-2}^{1}$$

$$= \left(2 - \tfrac{1}{2} - \tfrac{1}{3}\right) - \left(-4 - 2 + \tfrac{8}{3}\right)$$

$= \tfrac{9}{2}$ or 4.5

2

$$\boxed{A = \tfrac{9}{2} \text{ or } 4.5}$$

5. A wind turbine has to be brought to a stop for maintenance. At the time the brakes are initially applied, the turbine is rotating at 2 rads per second. During the braking, the angular acceleration of the turbine is given by

$$\alpha = -0.015\sqrt{1+5t}$$

where α is in rads/s^2 and t is in seconds. How long does it take for the turbine to come to a complete stop? (5 points)

(Hint: rotating objects have angular acceleration $\alpha = \dfrac{d\omega}{dt}$.)

$$\omega = \int \frac{d\omega}{dt}\, dt$$

$$= \int \alpha \, dt$$

$$= \int -0.015\sqrt{1+5t}\, dt$$

let $u = 1+5t$

$du = 5dt$

$\frac{du}{5} = dt$

$$= \int \frac{-0.015}{5} u^{1/2} du$$

$$= -0.003 \cdot \frac{2}{3} u^{3/2} + C$$

$$= -0.002 (1+5t)^{3/2} + C$$

at $t=0$, $\omega = 2$

$$2 = -0.002(1) + C$$

$$C = 2.002$$

and $\boxed{\omega = -0.002(1+5t)^{3/2} + 2.002}$

find t for $\omega = 0$:

$$0 = -0.002(1+5t)^{3/2} + 2.002$$

$$(1+5t)^{3/2} = \frac{2.002}{0.002}$$

$$1+5t = (1001)^{2/3}$$

$$t = \frac{1001^{2/3} - 1}{5}$$

$$\approx 19.8133$$

$\boxed{\approx 20 \text{ seconds}}$