Math 187 – Quiz #2

February 15, 2013 **Name:** Solution Set
Instructors: Patricia Wrean & Gilles Cazelais

Total: 25 points

Evaluate the following integrals.

1. $\int \frac{3x\,dx}{5-4x^2}$ (3 points)

let $u = 5-4x^2$
$du = -8x\,dx$.

$$= \int -\frac{3}{8}\frac{du}{u}$$

$$= -\frac{3}{8}\ln|u| + C$$

$$= -\frac{3}{8}\ln|5-4x^2| + C$$

2. $\int \frac{e^{2x}\,dx}{\sqrt{e^{2x}+5}}$ (3 points)

let $u = e^{2x}+5$
$du = 2e^{2x}\,dx$

$$= \int \frac{1}{2} u^{-1/2}\,du$$

$$= \frac{1}{2}\cdot\frac{2}{1}u^{1/2} + C$$

$$= \sqrt{e^{2x}+5} + C$$

3. $\int \sin^3\theta \, d\theta$ $= \int \sin\theta \,(1-\cos^2\theta)\, d\theta$ (1) (3 points)

$\boxed{\begin{aligned} &\text{let } u = \cos\theta \\ &du = -\sin\theta\, d\theta \end{aligned}}$ (1)

$= \int -(1-u^2)\, du$

$= \int (u^2-1)\, du$

$= \dfrac{u^3}{3} - u + C$

$= \dfrac{\cos^3\theta}{3} - \cos\theta + C$ (1)

4. $\int_0^{\pi/4} e^{\sin 2\theta} \cos 2\theta \, d\theta$ (3 points)

$\boxed{\begin{aligned} &\text{let } u = \sin 2\theta \\ &du = 2\cos 2\theta\, d\theta \end{aligned}}$ (1)

when $\theta = 0,\ u = 0$
$\theta = \pi/4,\ u = 1$

$= \int_0^1 \dfrac{e^u\, du}{2}$

$= \dfrac{e^u}{2} \Big|_0^1$ (1)

$= \frac{1}{2}(e^1 - e^0)$

$= \frac{1}{2}(e-1)$

$[\approx 0.859$ if you insist$]$ (1)

if said lower limit is zero, get 1.35914 instead (−1)

if $e^{\sin\frac{2\pi}{4}} \approx 1.03$ calculator in rad mode, answer is 0.0138

5. Find the arc length of the curve $y = \ln(\cos x)$ over the interval $0 \le x \le \frac{\pi}{4}$. Give an exact answer or round your answer to two decimal points. (3 points)

$$S = \int_a^b \sqrt{1 + (dy/dx)^2}\, dx$$ where $y = \ln(\cos x)$

$$\frac{dy}{dx} = \frac{1}{\cos x} \cdot (-\sin x) = -\tan x$$

and $\left(\frac{dy}{dx}\right)^2 = \tan^2 x$ 1

so
$$S = \int_0^{\pi/4} \sqrt{1 + \tan^2 x}\, dx$$
$$= \int_0^{\pi/4} \sqrt{\sec^2 x}\, dx$$
$$= \int_0^{\pi/4} \sec x\, dx$$ 1
$$= \ln|\sec x + \tan x| \Big|_0^{\pi/4}$$
$$= \ln\left|\frac{1}{\cos \pi/4} + \tan \pi/4\right| - \ln\left|\frac{1}{\cos 0} + \tan 0\right|$$
$$= \ln|\sqrt{2} + 1| - \ln 1$$

$$\boxed{S = \ln(\sqrt{2} + 1)}$$

$\Leftarrow$ since $\sqrt{2} + 1 > 0$, can drop absolute value signs if you wish

$$\approx 0.881$$

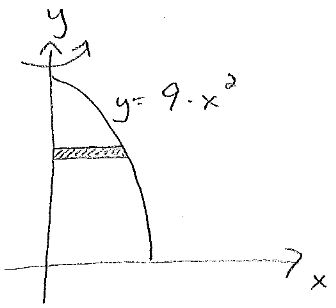
$$\boxed{S \approx 0.88}$$ 1

6. Consider the solid generated by revolving around the y-axis the first quadrant region bounded by $y = 9 - x^2$ and the x- and y-axes. (5 points)

   a) Calculate the volume of this solid. 2

   b) Locate the centroid of this solid. 3

   Give exact answers and/or approximations rounded to one decimal place.



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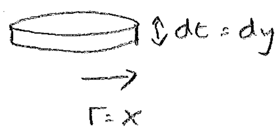


and $y = 9 - x^2$

$x^2 = 9 - y$

$dV = \pi r^2 dt$

$= \pi x^2 dy$

$= \pi (9 - y) dy$ 1

a) $V = \int_V dV$

$= \int_0^9 \pi (9-y)\, dy$

$= \pi \left(9y - \frac{y^2}{2}\right) \Big|_0^9$

$= \pi \left(81 - 81/2\right)$

$= \frac{81\pi}{2}$ 1

$\boxed{V = \frac{81\pi}{2}}$ $\boxed{\approx 127.235}$

b) $\bar{y} = \frac{1}{V} \int_V y_e \, dV$

$= \frac{2}{81\pi} \int_0^9 y \, \pi (9-y)\, dy$ 1

$= \frac{2}{81\cancel{\pi}} \cancel{\pi} \int_0^9 (9y - y^2)\, dy$

$= \frac{2}{81} \left(\frac{9y^2}{2} - \frac{y^3}{3}\right) \Big|_0^9$

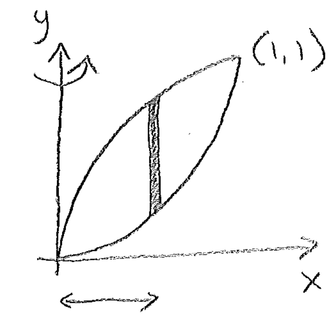
$= \frac{2}{81} \left(\frac{729}{2} - \frac{729}{3}\right)$

$= 3$ 1

by symmetry, $\bar{x} = 0$, so 1

$\boxed{\bar{x}, \bar{y} = (0, 3)}$

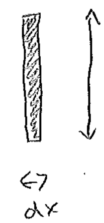
7. Find the moment of inertia with respect to the y-axis for the thin, flat, uniform plate in the shape of the region bounded by $y = x^2$ and $y = \sqrt{x}$. Give your answer in exact form or rounded to three decimal places. (5 points)



$r = x$

and $r^2 = x^2$ 1

Slice:



$\ell = y_2 - y_1 = \sqrt{x} - x^2$

$dA = \ell\, dx = (\sqrt{x} - x^2)\, dx$ 1

for thin uniform plate,

$$I = \rho t \int_A r^2\, dA$$

$$= \rho t \int_0^1 x^2 (\sqrt{x} - x^2)\, dx \qquad 1$$

$$= \rho t \int_0^1 (x^{5/2} - x^4)\, dx$$

$$= \rho t \left(\frac{2}{7} x^{7/5} - \frac{1}{5} x^5 \right) \Big|_0^1 \qquad 1$$

$$= \rho t \left(\frac{2}{7} - \frac{1}{5} \right)$$

$$\boxed{I = \frac{3}{35} \rho t}$$

$$\approx 0.085714\, \rho t$$

or $\boxed{I \approx 0.086\, \rho t}$ 1

If said $x^2 \cdot \sqrt{x} = x$, -1
will get $\frac{3}{10} \rho t$