Math 187 – Quiz #3

March 8, 2013 **Name:** Solution set
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Total: 25 points

Evaluate the following integrals.

1. $\int x\, e^{5x} dx$ (3 points)

method #1: table

D	I
x	e^{5x}
1	$\frac{1}{5}e^{5x}$
0	$\frac{1}{25}e^{5x}$

$(+)$ $(-)$

$$\boxed{= \tfrac{1}{5} x e^{5x} - \tfrac{1}{25} e^{5x} + C}$$

or

$$\frac{1}{25} e^{5x}(5x-1) + C$$

method #2:

$u = x \qquad v = \frac{1}{5}e^{5x}$

$du = dx \qquad dv = e^{5x} dx$

$$\int u\,dv = uv - \int v\,du$$

$$\int x e^{5x} dx = \frac{1}{5} x e^{5x} - \int \tfrac{1}{5} e^{5x} dx$$

$$= \tfrac{1}{5} x e^{5x} - \frac{1}{25} e^{5x} + C$$

2. $\int \frac{dy}{y^2 - 4y + 13}$ (3 points)

$$= \int \frac{dy}{(y-2)^2 + 3^2}$$

$$\boxed{= \frac{1}{3} \tan^{-1}\left(\frac{y-2}{3}\right) + C}$$

completing the square:

$$y^2 - 4y + 13 = y^2 - 4y + \underline{4} + 13 - \underline{4}$$

$$= y^2 - 4y + 4 + 9$$

$$= (y-2)^2 + 9$$

3. Use an appropriate trig substitution to evaluate (4 points)

$$\int \frac{dx}{x^2\sqrt{x^2-16}}$$

let $x = 4\sec\theta$

$dx = 4\sec\theta\tan\theta\, d\theta$

1/2

$$= \int \frac{4\sec\theta\tan\theta\,d\theta}{16\sec^2\theta\sqrt{16\sec^2\theta-16}}$$ 1/2

$$= \int \frac{4\sec\theta\tan\theta\,d\theta}{16\sec^2\theta\sqrt{16\tan^2\theta}}$$ 1/2

$$= \int \frac{\cancel{4}\,\cancel{\sec\theta}\,\cancel{\tan\theta}\,d\theta}{16\sec^{\cancel{2}}\theta\,\cancel{4}\,\cancel{\tan\theta}}$$

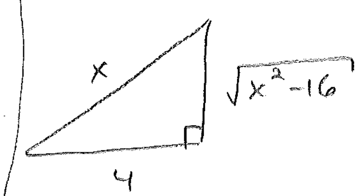
$$= \int \frac{1}{16\sec\theta}\,d\theta$$ 1

$$= \int \frac{\cos\theta}{16}\,d\theta$$

$$= \frac{\sin\theta}{16} + C$$ 1/2

$$\boxed{= \frac{\sqrt{x^2-16}}{16x} + C}$$ 1

$\sec\theta = x/4$



so $\sin\theta = \dfrac{\sqrt{x^2-16}}{x}$

4. Evaluate $\int \frac{x^2+x-12}{(x-1)(x^2+9)}\,dx$ (5 points)

partial fractions:

$$\frac{x^2+x-12}{(x-1)(x^2+9)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+9}$$ ½

$$x^2+x-12 = A(x^2+9) + (Bx+C)(x-1)$$ ½

set $x=1$: $-10 = 10A$

$A = -1$

set $x=0$: $-12 = 9A - C$

$-12 = -9 - C$

$C = 3$

set $x=2$: $-6 = 13A + 2B + C$

$-6 = -13 + 2B + 3$

$4 = 2B$

$B = 2$

1½

½

so $$\int \frac{x^2+x-12}{(x-1)(x^2+9)}\,dx = \int \left[\frac{-1}{x-1} + \frac{2x+3}{x^2+9}\right]dx$$

½ ½ ½

$$= \int \left[-\frac{1}{x-1} + \frac{2x}{x^2+9} + \frac{3}{x^2+9}\right]dx$$

$$\boxed{= -\ln|x-1| + \ln|x^2+9| + \tan^{-1}\frac{x}{3} + C}$$

can drop ||s if you wish

5. Find the first partial derivatives of the following function with respect to each of the independent variables. (2 points)

$f(r,t) = 4r^2 + r\ln\left(t^3\right)$ $= 4r^2 + 3r\ln t$

$$\boxed{\frac{\partial f}{\partial r} = 8r + \ln t^3 \qquad \frac{\partial f}{\partial t} = \frac{3r}{t}}$$

6. Evaluate. (3 points)

$\int_1^2 \int_0^{\sqrt{y}} \left(2xy + y^2\right) dx\, dy$

$= \int_1^2 \left(x^2 y + x y^2\right)\Big|_0^{\sqrt{y}} dy$

$= \int_1^2 \left(y \cdot y + \sqrt{y} \cdot y^2\right) dy$

$= \int_1^2 \left(y^2 + y^{5/2}\right) dy$

$= \left(\frac{y^3}{3} + \frac{2}{7} y^{7/2}\right)\Big|_1^2$

$= \left(\frac{8}{3} + \frac{2}{7} 2^{7/2}\right) - \left(\frac{1}{3} + \frac{2}{7}\right)$

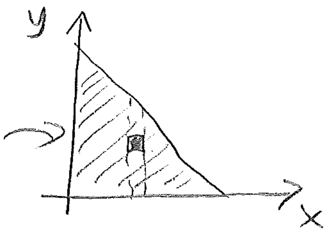
$$\boxed{= \frac{43}{21} + \frac{16\sqrt{2}}{7} \approx 5.28}$$

note: if said $y^2 y^{1/2} = y^1$, and $y^{5/2}$, get $23/6$ $\left(-\frac{1}{2}\right)$

if said $y^2 y^{1/2} = y^{3/2}$, get $\frac{29}{15} + \frac{8\sqrt{2}}{5}$ ≈ 4.196

7. Find the first-octant volume below the surface $z = 3x^2$ and bounded by the plane $x + y = 3$. (5 points)

region R



$dA = dx\,dy$ or $dy\,dx$

$$\left.\begin{array}{l} 0 \le x \le 3 \\ 0 \le y \le 3 - x \end{array}\right\} \text{(2)}$$

see next page for solution with limits switched

$$V = \iint_{\text{region } R} z \, dA$$

$$= \int_0^3 \int_0^{3-x} 3x^2 \, dy \, dx \qquad (1)$$

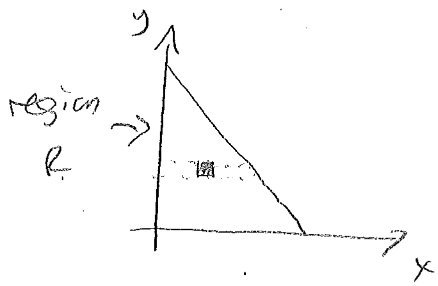
$$= \int_0^3 \left[3x^2 y \Big|_0^{3-x} \right] dx$$

$$= \int_0^3 3x^2(3-x) \, dx \qquad (1)$$

$$= \int_0^3 (9x^2 - 3x^3) \, dx$$

$$= \left(3x^3 - \frac{3}{4}x^4\right)\Big|_0^3$$

$$= 3 \cdot 27 - \frac{3 \cdot 81}{4} \quad \boxed{= \frac{81}{4} = 20.25} \qquad (1)$$



$dA = dx\,dy$ or $dy\,dx$

$0 \le y \le 3$

$0 \le x \le 3-y$

$$V = \iint\limits_{\text{region } R} z \, dA$$

$$= \int_0^3 \int_0^{3-y} 3x^2 \, dx \, dy$$

$$= \int_0^3 \left(x^3 \Big|_0^{3-y} \right) dy$$

$$= \int_0^3 (3-y)^3 \, dy$$

$$= -\frac{(3-y)^4}{4} \Big|_0^3$$

$$= 0 + \frac{3^4}{4} = \boxed{\frac{81}{4} = 20.25}$$